GSTGC 2018 Abstracts

1st Plenary Session
Saturday, 9:00 - 10:00

LCA A1
Coloring maps on surfaces
Ian Agol (University of California, Berkeley)
We'll discuss a question of whether a map on a surface has a finite-sheeted cover which is 4-colorable? We'll discuss various special cases of this question, and how it would follow from a dynamical conjecture.

1st Graduate Student Session
Saturday, 10:15 - 10:45

LCF F1
Associative and Cayley Grassmannians
Ustun Yildirim (Michigan State University)
In 1967, Brown and Gray showed that an exceptional (r-fold) cross product exists only for $r = 2$ in dimensions 3 and 7 and for $r = 3$ in dimensions 4 and 8. Using these exceptional cross products, we can identify special subvarieties of Grassmann varieties in dimensions 7 and 8. They are called associative and Cayley Grassmannian respectively. These Grassmannians are of natural interest in calibrated geometry: The tangent spaces of calibrated submanifolds (absolutely volume minimizing submanifolds in their homology classes) lie in these Grassmannians.

Over complex numbers, these Grassmannians are not compact. In this talk, after I talk about the necessary background, I will explain some of the results I obtained on the minimal compactification of Cayley Grassmannian.

LCF F3
Cycle index sum for non-k-equal configurations
Keely Grossnickle (Kansas State University)
I will present the cycle index sum of the symmetric group action on the homology of the configuration spaces of points in a Euclidean space with the condition that no k of them are equal. Configuration spaces form a bimodule structure over the little d-disc operad. This connection will be explained in my talk as well as the induced structure on the homology. (Joint work with Victor Turchin)

LCF F4
Holomorphic retracts of Teichmüller space
Dmitri Gekhtman (California Institute of Technology)
The Teichmüller space of a closed surface carries a natural complex structure, whose analytic properties reflect the topology and geometry of the surface. In this talk, we the discuss the problem of classifying the holomorphic retracts of Teichmüller space. Our approach hinges on the analysis of two dynamical flows - one in the moduli space of half-translation surfaces, and the other in the space of bounded holomorphic functions on the polydisk.

LCF F6
Hyperbolization on infinite type 3-manifolds
Tommaso Cremaschi (Boston College)
We will give an example of an open 3-manifold M that is locally hyperbolic, $\pi_1(M)$ has no divisible subgroup but M is not hyperbolic. This examples answers a question of Agol. We will use this example to explain a result on hyperbolaztion for a particular class of open 3-manifolds.

2nd Graduate Student Session
Saturday, 11:00 - 11:30

LCF F1
Positive Ricci curvature on manifolds with large torus actions
Diego Corro Tapia (Karlsruhe Institute of Technology)

For a simply-connected $(n+2)$-manifold M with a smooth effective action of the n-torus we construct an invariant Riemannian metric with positive Ricci curvature. Our work generalizes the construction of Bazaĭkin and Matvienko to all dimensions to get the following result:

> **Theorem:** If M is a smooth closed simply-connected $(n+2)$-manifold that admits an effective n-torus action, then M admits an equivariant smooth Riemannian metric of positive Ricci curvature.

By the work of Oh, our construction yields examples in all dimensions, and for M of dimension 4, 5, and 6 an explicit list of manifolds can be given.

LCF F3
Relating quotients by abelian and nonabelian groups
Rachel Webb (University of Michigan)

The abelian-nonabelian correspondence says that various invariants of the GIT quotient $X/\!\!/G$ should be related to those of $X/\!\!/T$, when X is a complex manifold, G is a reductive group, and T is a maximal torus in G. At the cohomology level this correspondence is well understood; the full correspondence of Gromov-Witten invariants for all X and G is still open. I will explain something of the cohomological relationship of $X/\!\!/G$ and $X/\!\!/T$ and some new results relating their Gromov-Witten theory, specifically their I-functions. The new results apply when X is a vector space and G is a connected group.

LCF F4
Application of combinatorial bordered Heegaard Floer
Rebeccah MacKinnon (University of Iowa)

Bordered Heegaard Floer homology is an invariant of three-manifolds with boundary. In particular Lipshitz, Ozsváth, and Thurston constructed invariants associated to a handle decomposition of a surface F, an algebra $\mathcal{A}(F)$, and an arc slide between two handle decompositions of F. In 2016, Zhan expanded on this work by constructing a combinatorial construction and proof of these invariants. In this entry level talk, we will give an overview of this combinatorial construction and discuss how we are leveraging it to prove an equivalence between tangle Floer homology and the contact category.

LCF F6
Cubulable quotients of free products
Ben Stucky (University of Oklahoma)

In 2015, Martin and Steenbock proved that a small cancellation quotient of a free product acts properly and cocompactly on a CAT[0] cube complex if the factors do, thus verifying a powerful non-positive curvature condition for these groups. To do this, they construct a nicely-behaved wallspace structure in the universal cover which builds upon the existing wallspace structures of the factors and invoke the Sageev construction. Shortly thereafter, Jankiewicz and Wise placed a special case of their results in the context of Wise's cubical small cancellation theory. I will describe work in progress seeking to cubulate some quotients of free products which are not necessarily small cancellation.

1st Young Faculty Session
Saturday, 11:45 - 12:45

LCF F1

Lifting curves on surfaces via 3-manifolds and the curve complex

Priyam Patel (University of California, Santa Barbara)

Given two simple closed curves α and β intersecting many times on an orientable surface S, we are interested in studying the minimal degree of a finite cover of S such that there is a lift of α disjoint from a lift of β. In joint work with Tarik Aougab and Sam Taylor, we use the geometry of hyperbolic 3-manifolds to obtain lower bounds on this degree in terms of curve complex distance between the curves α and β. After describing some of the techniques we use, I will highlight an interesting application of our work that gives lower bounds on the degrees of special covers for certain cube complexes associated to surfaces.

LCF F4

The fiber of the persistence map and other problems from TDA

Justin Curry (University at Albany, SUNY)

Algebraic topology provides discrete invariants of spaces—sets of components, homology groups, and so on. To make these invariants useful for studying naturally occurring examples in applications or data, one often needs to "persistify" these constructions by considering topological invariants in families, which leads to sheaves and cosheaves in a natural way. In this talk I'll start by describing persistent versions of connected components and homology called merge trees and barcodes. I'll also present a combinatorial result that describes explicitly how the space of merge trees fibers over the space of barcodes. Finally I point to the frontier of topological data analysis (TDA) and illustrate the need to use tools from analysis, algebra, geometry and topology all at once.

2nd Plenary Session
Saturday, 2:00 - 3:00

LCA A1

Geometric evolution equations

Natasa Sesum (Rutgers University)

We will discuss geometric evolutions equations such as the Ricci flow and the mean curvature flow. The emphasis will be on ancient solutions, that appear as singularity models, their properties and classification results.

3rd Graduate Student Session
Saturday, 3:15 - 3:45

LCF F1

Collapse via Cheeger deformations

Lawrence Mouillé (University of California, Riverside)

The Gromov–Hausdorff distance, which measures how far compact metric spaces are from being isometric, has proven to be useful in studying manifolds with lower curvature bounds. We say that a convergent sequence of spaces of equal dimension collapses if the Gromov–Hausdorff limit has strictly lower dimension. The structure of collapse of 3-dimensional spaces with non-negative curvature was instrumental in Perelman's proof of Thurston's geometrization conjecture, and hence the Poincaré conjecture. In the higher-dimensional case however, strikingly little is known.

In this talk, we will investigate the structure of collapse in the context of Cheeger deformation, a construction that collapses any manifold with an isometric action by a connected group to its orbit space. We will show that the highest-dimensional orbits become totally geodesic and normal homogeneous in the long-run [Searle–Solórzano–Wilhelm, 2015], and we will present current work towards understanding the structure of collapse near singular orbits.

LCF F3

Geometric finiteness in negatively pinched Hadamard manifolds

Beibei Liu (University of California, Davis)

We generalize Bonahon's characterization of geometrically infinite torsion-free discrete subgroups of $\mathrm{PSL}(2,\mathbb{C})$ to geometrically infinite discrete torsion-free subgroups Γ of isometries of negatively pinched Hadamard manifolds X. We then generalize a theorem of Bishop to prove that every such geometrically infinite isometry subgroup Γ has a set of nonconical limit points with cardinality of continuum.

LCF F4

Metric thickenings of Euclidean submanifolds

Joshua Mirth (Colorado State University)

Given a sample, X, from a manifold, M, how much of the topological and geometric structure of M is it possible to recover? One answer to this motivating question in applied topology is that the homotopy type of M can recovered by constructing a Vietoris–Rips or Čech simplicial complex with vertex set X. However, these simplicial complexes need not inherit the metric structure of the manifold.

We define *metric thickenings* of X, called the Vietoris–Rips and Čech thickenings, which are equipped with the 1-Wasserstein metric instead of the simplicial complex topology. We show that for Euclidean subsets M with positive reach the metric thickening of M is homotopy equivalent to M. In contrast to the classical theorem of Hausmann for simplicial complexes, our homotopy equivalence is given by canonical maps in both directions, is realized by linear homotopies from the map compositions to the corresponding identity maps, and is furthermore a deformation retraction.

LCF F6

Stable commutator length in generalized Baumslag–Solitar groups

Lvzhou Chen (University of Chicago)

The stable commutator length (scl) is a group invariant that measures the least topological complexity of surfaces bounding a given curve in the $K(G,1)$ space. A question essentially due to Gromov asks on which groups scl only takes rational values. This is challenging because scl is hard to compute. We show scl is rational in Baumslag–Solitar groups, and more generally graphs of cyclic groups, by giving an algorithm to compute scl of arbitrary elements using linear programming.

2nd Young Faculty Session

Saturday, 4:00 - 5:00

LCF F1

Surface group actions on complex flag varieties

Andrew Sanders (Heidelberg University)

Given a connected, oriented closed surface Σ of genus at least two, the study of the possible actions of $\pi_1(\Sigma)$ on compact complex homogeneous spaces G/H yields a rich universe in which the fields of algebra, geometry, topology and analysis become intertwined. Classically, the richest source of examples arose from studying the action of discrete subgroups $\Gamma < \mathrm{PSL}(2,\mathbb{C})$ (where $\Gamma \simeq \pi_1(\Sigma)$) on the complex projective line $\mathbb{CP}^1$.

In the past decade, many new examples of discrete subgroups $\Gamma < \mathrm{PSL}(n,\mathbb{C})$ have been found, and a natural question is to understand the geometry of these actions on various flag varieties associated to $\mathrm{PSL}(n,\mathbb{C})$.

In this talk, we will discuss the construction of domains of proper discontinuity Ω for these actions on flag varieties, and discuss the topology of the resulting manifolds $\Gamma\backslash\Omega$ which arise via the quotient of these domains. These questions are at the forefront of a flourishing field of research which is known as higher Teichmüller theory.

This talk will assume no prior knowledge of flag varieties, and a significant goal will be to explain the relevant parts of their geometry, and how they pertain to the question at hand.

LCF F4

Counting curves on a hyperbolic surface

Jenya Sapir (Binghamton University)

We will look at closed loops on a surface S of genus g. We choose a hyperbolic metric on S, and tighten each loop to a geodesic, so that it is as short as possible. For each real number L, there are finitely many geodesic loops of length at most L. We will talk about how this number grows with L. In particular, we will discuss Mirzakhani's famous result that the number of simple closed geodesics on S grows like L^{6g-6}. There will be lots of pictures.

4th Graduate Student Session

Sunday, 9:00 - 9:30

LCF F1

New examples of hidden symmetry free hyperbolic knots

Priyadip Mondal (University of Pittsburgh)

The hyperbolic knots having hidden symmetries are not classified yet. In fact, the two dodecahedral knots due to Aitchison and Rubinstein and the figure eight knot complete the list of hyperbolic knots not free of hidden symmetries with which we are familiar. In this talk, I will speak about a relation between the existence of hidden symmetries in hyperbolic knots corresponding to $1/n$ Dehn surgeries on an unknotted component of a two-component hyperbolic link and geometric isolation of the cusps of the link. I will also talk about a result which shows that if we start with a hyperbolic link with no more than nine crossings, then, there is a tail in the corresponding family of knots which will only contain hyperbolic knots free of hidden symmetries. This is joint work with Jason DeBlois.

LCF F3

The Heisenberg plane

Steve Trettel (University of California, Santa Barbara)

The Heisenberg group acts on the plane by translations and shears. The resulting geometry is the unique common degeneration of the constant curvature geometries $\mathbb{S}^2, \mathbb{E}^2, \mathbb{H}^2$ modeled within projective geometry, motivating the study of Heisenberg surfaces and regenerations. In this talk we will discuss the classification of Heisenberg structures on orbifolds and their deformation spaces.

LCF F4

Degeneration of abelian differentials and period matrices

Xuntao Hu (Stony Brook University)

The plumbing parameters give local coordinates near the boundary of the Deligne–Mumford compactification of the moduli spaces of curves. In this talk we introduce a new method to construct smooth abelian differentials near an arbitrary boundary strata. This method further allows us to compute the variational formula of the degenerating family of abelian differentials in terms of the plumbing parameters. We also give the variational formula for the degeneration of the period matrices, generalizing results of Yamada and Taniguchi. This is a collaborate work with Chaya Norton.

LCF F6

An application of Salem numbers to braid group representations

Nancy Scherich (University of California, Santa Barbara)

Many well known braid group representations have a parameter. I will show how to carefully choose evaluations of the parameter to force the representation to be discrete, and sometimes even land in a lattice. It is surprising and exciting to see how careful algebraic constructions can lead to geometric results.

5th Graduate Student Session
Sunday, 9:45 - 10:15

LCF F1
Characteristic numbers of manifold bundles over surfaces with highly connected fibers
Jens Reinhold (Stanford University)

A theorem of Hambleton–Korzeniewski–Ranicki implies that the signature of any closed $4m$-manifold that fibers over a surface is divisible by 4. However, one may ask if restricting the diffeomorphism type of the fiber gives stronger divisibility constraints. I will present an overview of work in progress, joint with M. Krannich, on the group of oriented cobordism classes in dimension $4m$ that arise as total spaces of fiber bundles over a surface with fiber a highly connected manifold. Examples of such fibers are the manifolds $W_g = \#^g(S^{2m-1} \times S^{2m-1})$. As a corollary, it is possible to determine all signatures of manifolds that fiber over a surface with fiber W_g, if $g \geq 7$. It turns out that signature 4 is possible if and only if m is one of 1, 2 or 4.

LCF F3
Symmetric spaces as moduli spaces
Mark Greenfield (University of Michigan)

Using a modular interpretation of the symmetric space $\mathrm{SL}(n, \mathbb{R})/\mathrm{SO}(n)$, we study its geometry and compactification. In particular, it can be viewed as the Teichmüller space of flat tori of dimension n. This provides a natural generalization of the well-known Teichmüller space of the 2-torus, which is realized as the basic symmetric space $\mathbb{H}^2$, the Poincaré upper half-plane. Once the spaces are identified, we will explore a few different metrics on them. It will turn out that in dimensions 3 and higher, the situation is a bit more complicated, but leads to many more questions. This is based on joint work with Lizhen Ji.

LCF F4
What is a synthetic spectrum?
Piotr Pstrągowski (Northwestern University)

A basic invariant of a space or a spectrum X is its homology E_*X, which often carries a structure of a so called comodule. The homological algebra of comodules informs our understanding of stable phenomena in algebraic topology, for example, it forms the second page of the Adams spectral sequence.

In this talk, we describe how to any reasonable homology theory one can associate a notion of a synthetic spectrum, this is a kind of a sheaf on the site of finite spectra with projective E-homology. We show that synthetic spectra exhibit both algebraic and topological features at once, intertwined in a non-trivial way. This talk is aimed at a general topological audience and will have expository character.

LCF F6
Equidistribution and counting in Hilbert geometry
Feng Zhu (University of Michigan)

Strictly convex Hilbert geometries are a sort of generalization of negatively-curved Riemannian geometry, and are examples of interest to those studying geometric structures and higher Teichmüller theory. Just like in negative curvature, these geometries are associated with dynamically interesting geodesic flows, and equidistribution and counting theorems that hold for negatively-curved spaces can be demonstrated for these Hilbert geometries as well. In this talk I will motivate and describe one set of such results.

LCF F1

A lower bound for the double slice genus

Wenzhao Chen (Michigan State University)

Every knot in the 3-sphere can be realized as a cross-section of some unknotted surface in the 4-sphere. For a given knot, the least genus of all such surfaces is defined to be its double slice genus. Obviously twice the slice genus of a knot is a lower bound for its double slice genus. One really basic question is whether the double slice genus can be arbitrarily large compared to twice the slice genus. However, this was not answered due to the lack of lower bounds for the double slice genus. In this talk I will introduce a lower bound that can be used to answer this question.

LCF F3

Spherical metrics with conical singularities on 2-spheres

Subhadip Dey (University of California, Davis)

Let $\theta_1, \theta_2, \ldots, \theta_n$ be positive numbers; we want to know whether there exists a spherical metric (i.e. a metric with constant curvature 1) on $\mathbb{S}^2$ with n *conical* singularities of angles $2\pi\theta_1, 2\pi\theta_2, \ldots, 2\pi\theta_n$. When $n = 1$, the answer is trivial for simple reasons. When $n = 2, 3$, the answer has been known for many years thanks to the works of Marc Troyanov ($n = 2$) and Alexandre Eremenko ($n = 3$). In recent years, a lot of progress has been made to answer this question in the general case ($n \geq 4$), starting with a recent breakthrough by Gabriele Mondello and Dmitri Panov. They obtained very strong sufficient conditions and necessary conditions on θ_i's; however, these conditions were not exact. In a recent work, we are able to strengthen these conditions by showing that their (sufficient) condition is also necessary when we assume that none of the θ_i's are integers. In this talk, I will highlight on these and related results.

LCF F4

A spectral sequence for Dehn fillings

Oliver Wang (Cornell University)

Given a group G and a collection $\mathcal{P}$ of subgroups, one can define the relative group cohomology $H^i(G, \mathcal{P}; \mathbb{Z}G)$. These cohomology groups are of interest when $(G, \mathcal{P})$ is relatively hyperbolic; in recent work with Jason Manning, it is shown that under reasonable assumptions, $H^i(G, \mathcal{P}; \mathbb{Z}G) \cong \check{H}^{i-1}(\partial(G, \mathcal{P}); \mathbb{Z})$ where $\partial(G, \mathcal{P})$ is the Bowditch boundary. Dehn filling is a process by which one can obtain relatively hyperbolic groups $(\bar{G}, \bar{\mathcal{P}})$ from a given relatively hyperbolic group $(G, \mathcal{P})$. In this talk, I will introduce a spectral sequence relating the cohomology groups $H^i(G, \mathcal{P}; \mathbb{Z}G)$ and $H^i(\bar{G}, \bar{\mathcal{P}}; \mathbb{Z}\bar{G})$. I will also discuss some consequences of this spectral sequence.

LCF F6

Sliding window embeddings of quasiperiodic functions

Hitesh Gakhar (Michigan State University)

Classically, Sliding Window Embeddings were used in the study of dynamical systems to reconstruct topology of underlying attractors from generic observation functions. In 2015, Perea and Harer studied persistent homology of sliding window embeddings from $\mathcal{L}^2$ periodic functions. We define a quasiperiodic function as a superposition of periodic functions with incommensurate frequencies. As it turns out, sliding window embeddings of quasiperiodic functions are dense in high dimensional torii. In this talk, I will present a strategy to study their persistent homologies.

3rd Young Faculty Session
Sunday, 11:15 - 12:15

LCF F1
On minimizers and critical points for anisotropic isoperimetric problems
Robin Neumayer (Northwestern University)

Anisotropic surface energies are a natural generalization of the perimeter that arise in models for equilibrium shapes of crystals. We discuss some recent results for anisotropic isoperimetric problems concerning the strong quantitative stability of minimizers, bubbling phenomena for critical points, and a weak Alexandrov theorem for non-smooth anisotropies. Part of this talk is based on joint work with Delgadino, Maggi, and Mihaila.

LCF F4
Geometry and dynamics of free group automorphisms, and their connections to low dimensional topology
Caglar Uyanik (Vanderbilt University)

I will talk about how the hyperbolic geometry of 3-manifolds and mapping class groups of surfaces provide a roadmap for understanding the geometry and dynamics of free group automorphisms. In particular, I will illustrate a Nielsen-Thurston type approach for studying the geometry of free group automorphisms and explain recent progress on this program.

3rd Plenary Session
Sunday, 1:00 - 2:00

LCA A1
Quantitative aspects of topology
Shmuel Weinberger (University of Chicago)

The most burning questions in topology are yes and no questions; is this manifold a sphere, does that manifold bound? However, merely knowing that something exists should leave one unsettled if one can't afterwards *see* it. I will discuss in two prototypical (and contrasting) cases from the last century (early important successes of the "algebraicization of geometric topology"), the issue of trying to understand how complicated the objects whose existences are inferred really are.

7th Graduate Student Session
Sunday, 2:15 - 2:45

LCF F1
The dimension of the restricted Hitchin component for triangle groups
Elise Weir (University of Tennessee, Knoxville)

A triangle group $T(p,q,r)$ is the group of rotational symmetries of a tiling of the hyperbolic plane by geodesic triangles. For representations in $PSL(3,\mathbb{R})$, the dimension of the Hitchin component for hyperbolic triangle groups follows from a special case of work by Choi and Goldman in 2005. More recently, Long and Thistlethwaite determined its dimension for general $n \geq 3$. We will establish results which retain this broader n-dimensional context, but focus in on those representations contained in particular subgroups of $PSL(n,\mathbb{R})$. As time permits, we will also discuss one motivation for studying these triangle groups– namely, the surface subgroups that they contain.

LCF F3
Random Coxeter groups
Angelica Deibel (Brandeis University)

The nerve of a Coxeter group is a simplicial complex whose homology gives some information about the cohomology of the group. In this talk, I will introduce random Coxeter groups and give some results about the homology of the nerve of a random Coxeter group.

LCF F4

Stable limits of polygon spaces

Jack Love (George Mason University)

Given a weight vector of positive real numbers $\ell = (\ell_1, \ldots, \ell_n) \in \mathbb{R}^n_{\geq 0}$, an ℓ-gon in $\mathbb{R}^d$ is an n-tuple of unit vectors in $\mathbb{R}^d$ whose weighted sum is 0. We call such a collection an ℓ-gon because placing the weighted unit vectors head-to-tail in $\mathbb{R}^d$ produces a closed linkage of line segments, i.e., a polygon, with side-lengths ℓ_i. For fixed ℓ, the collection of all ℓ-gons in $\mathbb{R}^d$ is denoted $E_\ell(d)$ and has a natural action of the special orthogonal group $SO(d)$. In this talk we explore the topological and algebraic properties of the orbit spaces $E_\ell(d)/SO(d)$ as ℓ varies in $\mathbb{R}^n$ and as the ambient space $\mathbb{R}^d$ increases in dimension. In particular we will look at "wall-crossing" phenomena as ℓ varies through a polyhedral complex in $\mathbb{R}^n$, and we show that for fixed ℓ the orbit spaces $E_\ell(d)/SO(d)$ stabilize at $d = n$.

LCF F6

Irreducible embeddings and equivariant twisted fiber sums

Andrew Havens (University of Massachusetts Amherst)

This talk will explore the possibility of constructing interesting surface embeddings in 4-manifolds by studying involutions on 4-manifolds. Drawing on the work of Finashin, Kreck and Viro in the 1980's, as well as recent work of Nermin Salepci on real Lefschetz fibrations, we will discuss methods to try and construct families of surfaces in the 4-sphere which are embedded homeomorphically, but are not smoothly isotopic. A further potential application of interest is being able to study involutions on 4-manifolds in conjunction with the symplectic geography problem.