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MATH 180 Final Exam
December 8, 2016

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: ____________________

University Email: ____________________

UIN: ____________________

Circle your instructor's name: Boester Dumas Embers Riedl Shulman

TA's Name: ____________________

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- You must show your work on all problems.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. (19 points) Find the following derivatives. YOU DO NOT NEED TO SIMPLIFY YOUR ANSWERS.

   (a) $\dfrac{d}{dx}\left(\dfrac{\cos x}{\ln x}\right)$

   (b) $\dfrac{d}{dx}\left(\arctan(x^2) \cdot e^x\right)$

   (c) $\dfrac{d}{dx}\left(\displaystyle\int_0^{x^3} t \cdot 3^t \, dt\right)$

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2. (24 points) Find the integrals below. Make sure to justify your work.

(a) $\displaystyle\int 4e^{3x}\ dx$

(b) $\displaystyle\int \left(\frac{1}{x^3} - \frac{1}{\sqrt{1-x^2}}\right)\ dx$

(c) $\displaystyle\int_1^e \left(\frac{1}{x} - 5\right)\ dx$

(d) $\displaystyle\int_{-3}^{3} x^5 \cos x\ dx$

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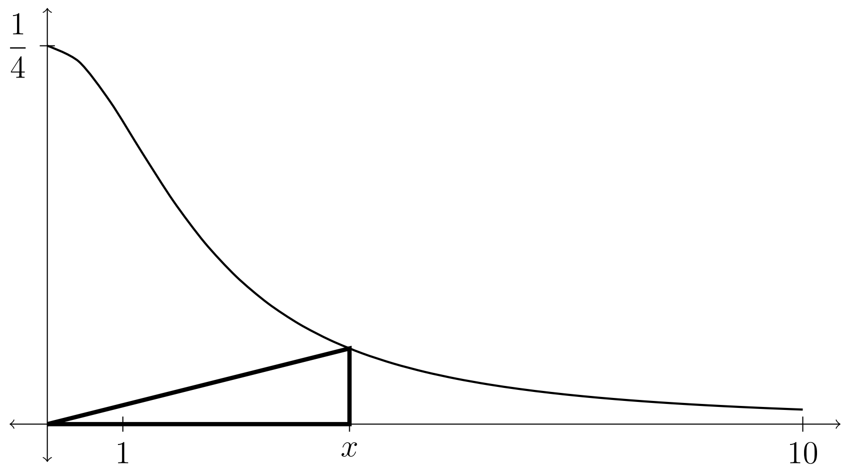
3. (20 points) Calculate each limit (by any method). Be sure to justify any method you use. If the limit does not exist, then you are required to determine whether the limit is $+\infty$ or $-\infty$.

   (a) $\displaystyle\lim_{x\to+\infty} \frac{\cos x}{\sqrt{x^2-1}}$

   (b) $\displaystyle\lim_{x\to-\infty} x^3 e^{-4x}$

   (c) $\displaystyle\lim_{x\to 1} \frac{3x^3-12x^2+15x-6}{2x^3-6x+4}$

4. (15 points) A right triangle has one vertex at the origin, a second vertex on the positive x-axis at $(x, 0)$, and a third vertex on the curve $y = \dfrac{1}{x^2+4}$ (see the figure below). If the point $(x, 0)$ satisfies $1 \le x \le 10$, what is the maximum and minimum area for such a triangle? Write your maximum and minimum areas in the space provided.



Minimum Area: ____________________ Maximum Area: ____________________

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5. (10 points) Consider a function $f(x)$ whose first and second derivatives are $f'(x) = \dfrac{x^2-9}{x^2+9}$ and $f''(x) = \dfrac{36x}{(x^2+9)^2}$. Make sure to justify your answers with a sign diagram or some other work.

(a) In the spaces provided, state the intervals where $f(x)$ is decreasing and those where $f(x)$ is increasing.

Decreasing: ______________________ Increasing: ______________________

(b) Find the critical points of $f(x)$ and classify each as a local minimum, local maximum, or neither.

(c) In the spaces provided, state the intervals where $f(x)$ is concave down and those where $f(x)$ is concave up.

Concave Down: ______________________ Concave Up: ______________________

(d) List any inflection points.

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6. (8 points) Consider the function $f(x) = x^2$ on the closed interval $[1, 4]$. Using the theorems below, complete each of the statements in (a)-(d). Each theorem is used at most once.

- Mean Value Theorem for Integrals
- Extreme Value Theorem
- Intermediate Value Theorem
- Mean Value Theorem
- Rolle's Theorem

(a) The ______________________________ says that there is a c in $(1, 4)$ with $f(c) = k$ where k is any number between 1 and 16.

(b) The ______________________________ says that there is a c in $(1, 4)$ with $f'(c) = \dfrac{16-1}{4-1}$.

(c) The ______________________________ says that there is a c in $(1, 4)$ with $f(c) = \dfrac{\frac{4^3}{3} - \frac{1^3}{3}}{4-1}$.

(d) The ______________________________ says that there is an absolute maximum and absolute minimum for $f(x)$ on $[1, 4]$.

7. (10 points) Consider the vectors $\mathbf{v} = \langle 2, 1\rangle$ and $\mathbf{w} = \langle -1, 1\rangle$. In the grid below, answer "YES" or "NO". In the first column answer if the given vector is PARALLEL to $\mathbf{v}$, and in the second column answer if the given vector is ORTHOGONAL to $\mathbf{w}$.

Given Vector	PARALLEL to $\mathbf{v}$?	ORTHOGONAL to $\mathbf{w}$?
$3\mathbf{v}$		
the projection of $\mathbf{u}$ on $\mathbf{v}$, $\text{proj}_{\mathbf{v}}(\mathbf{u})$		
the vector 45 degrees north of east with magnitude 4		
$4\mathbf{v} + 2\mathbf{w}$		
$\langle -3, -\frac{3}{2}\rangle$		

8. (12 points) Consider the function $f(x) = \sqrt{x}$.

   (a) Find the linear function that best approximates $f(x)$ at $a = 100$.

   (b) Using your answer to part (a), estimate $\sqrt{102}$.

   (c) Is your estimate in part (b) an overestimate or underestimate to the exact value of $\sqrt{102}$? Justify your answer.

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9. (12 points) Consider the function $f(x) = \dfrac{1}{1+x^2}$

(a) Calculate the left hand Riemann sum for $f(x)$ on $[0, 2]$ with 4 rectangles. Write your final answer as a reduced fraction.

(b) Is your Riemann sum in part (a) an overestimate or an underestimate to the actual area? Justify your answer.

(c) Find the actual area on $[0, 2]$. You may leave your answer unsimplified.

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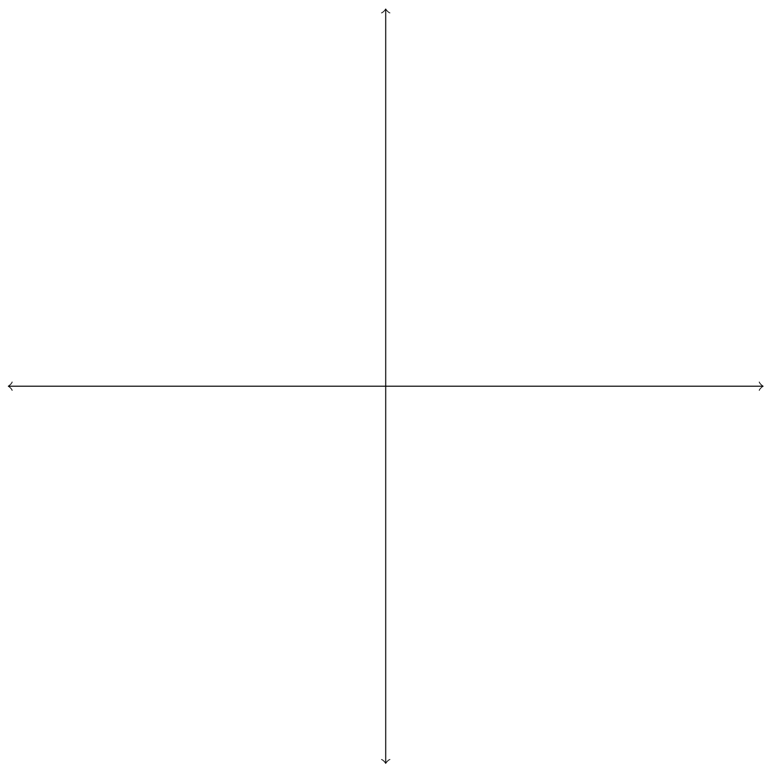
10. (10 points) Consider the points $A(1,1)$, $B(3,5)$, and $C(-3,3)$.

(a) Write the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in the form $\langle a, b\rangle$. Then sketch and label both of them on the axes provided.



(b) Calculate the magnitude of $\overrightarrow{AB}$ and the magnitude of $\overrightarrow{AC}$.

(c) Find the angle θ between $\overrightarrow{AB}$ and $\overrightarrow{AC}$. You can leave θ unsimplified.

11. (10 points) Consider the curve defined by

$$x\cos(y) + 4y = 4\pi.$$

Write the equation of the tangent line to the curve at the point $(0, \pi)$.

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