

MATH 180 Final Exam
December 9, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. **YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR.** This exam contains 14 pages (including this cover page) and 16 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty. Please put your initials on each page.

Your Name:_____

Your UIN:_____

UIC Email:_____

TA Name:_____

Proficiency Exam

Circle your instructor.

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| <ul style="list-style-type: none">• Martina Bode• Shavila Devi• Vi Diep | <ul style="list-style-type: none">• Matthew Lee• John Steenbergen |
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The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

1. (15 points) MULTIPLE CHOICE

Circle the correct answer.

- (a) (3 points) If
- $2 - x^2 \leq f(x) \leq -2x + 3$
- for all
- x
- , then by the Squeeze Theorem evaluate
- $\lim_{x \rightarrow 1} f(x)$
- .

0 1 2 None of these answers

- (b) (3 points) A function
- f
- is defined by

$$f(x) = \begin{cases} 2c & \text{if } x < 0 \\ 4 \cos x & \text{if } x \geq 0 \end{cases}$$

 f is continuous on $(-\infty, \infty)$ for $c = ?$

0 1 2 None of these answers

- (c) (3 points) Evaluate
- $\frac{d}{dx} \left(\int_0^{x^3} (\sin^3 t) dt \right)$
- .

$\sin^3(x^3)$ $\cos^3(3x^2)$ $\sin^3(3x^2)$ $3x^2 \cos^3(x^3)$ $3x^2 \sin^3(x^3)$

- (d) (3 points) Given that
- $f(2) = 4$
- , and
- $f'(x) \leq -2$
- for all
- x
- , can
- $f(3) = 5$
- ?

Yes No

- (e) (3 points) Suppose
- f
- is a continuous function. If
- $\int_2^9 f(x) = 7$
- then by the Mean Value Theorem for Integrals there is a
- c
- in
- $(2, 9)$
- with
- $f(c) = ?$

1 7 14 None of these answers

2. (20 points) Compute the following limits, use L'Hôpital's Rule if needed. If a limit is $+\infty$ or $-\infty$ state which one, and if the limit does not exist, explain why.

(a) (6 points) $\lim_{x \rightarrow -3} \frac{\sqrt{x+7} - 2}{x+3}$

(b) (6 points) $\lim_{x \rightarrow 1^-} \frac{x^3 + 1}{x^3 - 1}$

(c) (8 points) $\lim_{x \rightarrow 0^+} (1 + 2x)^{3/x}$

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3. (10 points) Use the **limit definition of derivatives** to find $f'(x)$ for the function $f(x) = 4x^2 - 5$. No credit will be given for using any other method.

4. (20 points) Differentiate the following functions, using logarithmic differentiation if needed. Do not simplify your answers.

(a) (6 points) $f(x) = x^2 \arctan(4x)$

(b) (6 points) $g(x) = \frac{e^{2x}}{x^2 + 1}$

(c) (8 points) $h(x) = (2 + \sin x)^{3x}$

5. (10 points) Find an equation of the tangent line to the graph of the function $f(x) = x + \sqrt{x}$ at $x = 1$.

6. (10 points) Find the slope of the tangent line to $xy + \cos y + x^2 = 10$ at the point $(3, 0)$.

7. (12 points) Consider a function $f(x)$ whose first and second derivatives are:

$$f'(x) = \frac{x^2 - 9}{x^2 + 1} \text{ and } f''(x) = \frac{20x}{(x^2 + 1)^2}$$

Make sure to justify your answers with a sign diagram or some other work.

- (a) (4 points) State the intervals where $f(x)$ is decreasing and those where $f(x)$ is increasing.

- (b) (2 points) Find the x -values of the critical points of $f(x)$ and classify each as a local minimum, local maximum, or neither.

- (c) (4 points) State the intervals where $f(x)$ is concave up and those where $f(x)$ is concave down.

- (d) (2 points) Find the x -values of inflection points, if any.

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8. (12 points) Consider $f(x) = x^3 - 12x + 4$. Find the absolute minimum and maximum value of f on the interval $[-3, 1]$. Show all your work.

9. (12 points) A kite 40 ft above the ground moves horizontally at a speed of 10 ft/s. At what rate is the string being released when 50 ft of the string is out? Show all your work.

10. (14 points) Find the point(s) on the parabola $y = \frac{1}{2}x^2$ that are closest to the point $(4, 1)$.

(a) (1 point) In words, what quantity (objective) needs to be optimized?

(b) (2 points) Draw a picture modeling this situation.

(c) (4 points) Find a function for the quantity that is being optimized, and find its domain.

(d) (3 points) Find the derivative of your function, and find the critical point(s).

(e) (4 points) Simplify and justify your answer. The final answer is:

11. (6 points) Given $\int_2^3 2f(x) dx = 4$ and $\int_3^0 f(x) dx = 3$, evaluate $\int_0^2 f(x) dx$.

12. (8 points) Consider the function $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 5$.

(a) (4 points) Sketch the graph indicating the left hand Riemann Sum L_2 for $n = 2$ and $f(x)$ on the interval $[1, 5]$. Compute its sum. Write your final answer as reduced fraction.

(b) (4 points) Find the actual area on $[1, 5]$. You can leave the answer in the exact form.

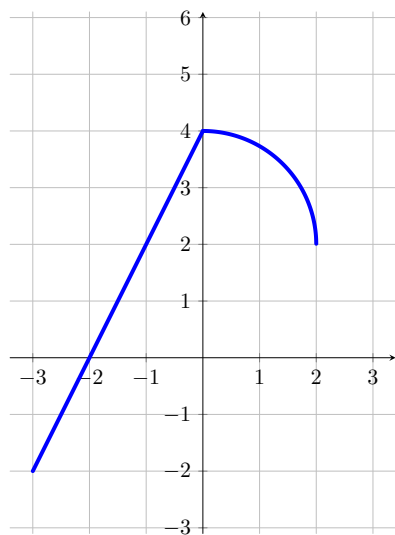
13. (12 points) Find the following definite integrals, no need to simplify your answers.

(a) (6 points) $\int_0^1 (2x^2 - 5x + e^x) dx$

(b) (6 points) $\int_{-\pi/4}^{\pi/4} (\cos x - 4 \sin(2x)) dx$

14. (14 points) Use the graph of $f(x)$, provided below, to define the area function $A(x) = \int_{-3}^x f(x) dx$ on the interval $[-3, 2]$.

(Assume that the graph of f consists of a line segment and a semi-circle)



- (a) (5 points) Evaluate $\int_{-3}^0 f(x) dx$, $\int_0^2 f(x) dx$, and $\int_{-3}^2 f(x) dx$.

- (b) (5 points) Evaluate $A(-3)$, $A(0)$, $A(2)$ and $A'(-1)$.

- (c) (4 points) On which intervals is A increasing? decreasing?

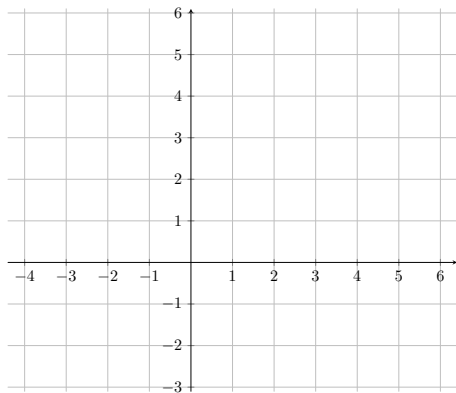
15. (18 points) Given $\mathbf{v} = \langle -1, 3 \rangle$ and $\mathbf{w} = \langle 4, -2 \rangle$.

(a) (4 points) Find the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$. Do not simplify!

(b) (4 points) Find a unit vector pointing in the same direction of the vector $\mathbf{v}$.

(c) (4 points) Find $\text{proj}_{\mathbf{v}} \mathbf{w}$.

(d) (6 points) Sketch the vectors $\mathbf{v}$, $\mathbf{w}$ and $\text{proj}_{\mathbf{v}} \mathbf{w}$.



16. (7 points) A tow truck pulls a van along a road. The chain makes an angle of $\pi/6$ with the road and the force of the chain is 500 N. How much work is done in towing the van 20 m?

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PAPER, BUT NOTHING ON THIS PAGE WILL BE GRADED.