Math 180 **Name (Print):** ______________________________

5/5/2016 **NetID:** ______________________________

Time Limit: 120 Minutes

This exam contains 14 pages (including this cover page) and 16 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on the top of this page.

The following rules apply:

- You may not open this exam until you are instructed to do so.
- You are expected to abide by the University's rules concerning Academic Honesty.
- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

TA Name:____________________

Circle your instructor.

- Martina Bode
- Jenny Ross
- Michael Hull
- Evangelos Kobotis
- Bonnie Saunders

1. (15 points) Suppose Leela is the first female astronaut to land on the moon. On the moon the acceleration due to gravity is -2 meter/sec^2. Leela jumps into the air with an initial velocity of 10 meter/sec.

(a) (6 points) What function describes her velocity?

$\begin{cases} a = -2 \\ v_0 = 10 \\ s_0 = 0 \end{cases}$

$v = -2t + c$ @ $t = 0$ $-2 \cdot 0 + c = 10 \Rightarrow c = 10$

$\boxed{v = -2t + 10}$

(b) (6 points) What function describes her height?

$\begin{cases} v = -2t + 10 \\ s_0 = 0 \end{cases}$ $\Rightarrow s = -t^2 + 10t + d$ @ $t = 0$ $0 + 0 + d = 0 \Rightarrow d = 0$

$\boxed{s = -t^2 + 10t}$

(c) (3 points) At what time does Leela reach her highest point?

@ highest point $v = 0$

$-2t + 10 = 0$

$\boxed{t = 5 \text{ seconds}}$

2. (13 points) Evaluate the following limits. State DNE, ∞ or $-\infty$, as appropriate if the limit does not exist. Clearly explain your reasoning, stating theorems as needed.

(a) (3 points) $\lim_{x \to 1} \frac{x^2 - 1}{x + 3}$ of type $\frac{0}{4} = 0$

$= \boxed{0}$

(b) (5 points) $\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x - 3}$ of type $\frac{0}{0}$, use the conjugate

$= \lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2}$

$= \lim_{x \to 3} \frac{\cancel{x+1-4}}{\cancel{x-3}} \cdot \frac{1}{\sqrt{x+1} + 2} = \frac{1}{\sqrt{4} + 2} = \boxed{\frac{1}{4}}$

(c) (5 points) $\lim_{x \to 0} \frac{12 - 12\cos x}{x^2}$ of type $\frac{0}{0}$, use L'Hôpital

$= \lim_{x \to 0} \frac{12 \sin x}{2x}$ of type $\frac{0}{0}$, use L'Hôpital again

$= \lim_{x \to 0} \frac{12 \cos x}{2} = \frac{12}{2} = \boxed{6}$

3. (5 points) Evaluate $\lim_{x\to 1^-} \frac{x^2-1}{x^2-x-2}$. of type $\frac{0}{-2} = 0$

$= \boxed{0}$

4. (10 points) Use the limit definition to compute the derivative function $f'(x)$ for the function $f(x) = 3x^2 - 5$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(3\cdot(x+h)^2 - \cancel{5}) - (3x^2 - \cancel{5})}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) - 3x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h}$$

$$= \lim_{h\to 0} 6x + 3h$$

$$= \boxed{6x}$$

5. (12 points) Find the derivatives of the following functions. You do not need to simplify your answers.

(a) (6 points) $g(x) = \tan^{-1}(e^x)$

chain rule

$$g'(x) = \frac{1}{1+(e^x)^2} \cdot e^x = \frac{e^x}{1+e^{2x}}$$

(b) (6 points) $s(t) = \dfrac{\sin t}{2\sqrt{t}+1}$

quotient rule

$u = \sin t$ $\quad v = 2\sqrt{t}+1$

$u' = \cos t$ $\quad v' = \frac{1}{\sqrt{t}} + 1$

$$s'(t) = \frac{\cos t\,(2\sqrt{t}+1) - \sin t\left(\frac{1}{\sqrt{t}}+1\right)}{(2\sqrt{t}+1)^2}$$

6. (12 points) Consider the function, $f(x) = \sqrt{x}$.

(a) (6 points) Find the linear approximation to f at the point $a = 9$.

$L(x) = f(9) + f'(9)(x-9)$

$f(9) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

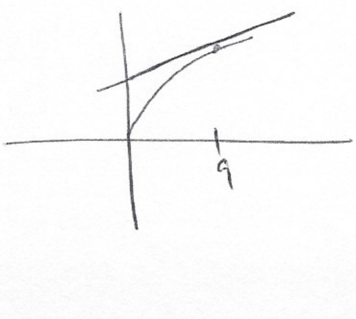
$\boxed{L(x) = 3 + \frac{1}{6}(x-9)}$

(b) (4 points) Use your answer to approximate $\sqrt{9.66}$ (give at least 2 decimal places in your answer).

$\sqrt{9.66} \approx L(9.66) = 3 + \frac{1}{6}(9.66 - 9) = 3 + \frac{1}{6} \cdot (.66)$

$= 3 + .11 = \boxed{3.11}$

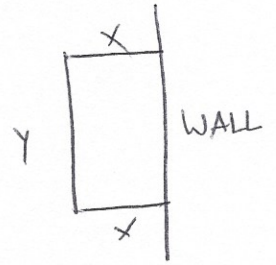
(c) (2 points) Is your solution an over or under estimate? Explain.



It is an over estimate because the tangent line is above the graph.

7. (15 points) You have 100 feet of fencing and want to enclose a rectangular area up against a long straight wall. Find the largest area that can be enclosed by following the steps below.

(a) (3 points) Sketch a diagram of the problem, define variables to be used and carefully label your picture.



$X \geq 0$ $\quad$ & $2x + y = 100$

$y \geq 0$

(b) (6 points) Find the optimization function. Include the domain of the function.

Area $= X \cdot y$ $\quad$ Subject to $2X + y = 100$ $\Rightarrow y = 100 - 2x$

$f(x) = x \cdot (100 - 2x) = \boxed{100x - 2x^2}$ $\quad$ $\boxed{\text{Domain} = [0, 50]}$

$x \geq 0$ & $y \geq 0$ $\Rightarrow$ $y = 100 - 2x \geq 0$

$\Rightarrow x \leq 50$

(c) (4 points) Find the dimensions that enclose the largest area.

$f'(x) = 100 - 4x$

(i) $f' = 0 \Rightarrow x = 25$

(ii) $f' = \phi$ N/A

(iii) endpoints $x = 0$ & $x = 50$

Closed Interval Method

	area
0	0
25	$25 \cdot 50 = 1250 = \max$
50	0

$\boxed{\begin{array}{c}\text{Dimensions} \\ x = 25 \\ y = 50\end{array}}$

(d) (2 points) Verify that your result is the maximum possible value.

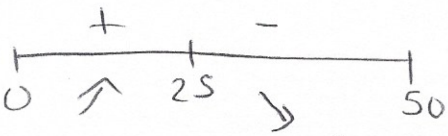
See (c) or first derivative method

$f' = 100 - 4x$

$f' = 0 \quad x = 25$

$f' = \phi$ N/A

$\Big\}$ Sign of f'



absolute max at

$x = 25$

8. (12 points) Circle the correct answer.

(a) If f is a continuous function on $[1,5]$ such that $f(1)=5, f(5)=2$, then by the Intermediate Value Theorem there is a c in $(1,5)$ with $f(c)=1$.

by IVT all values between
2 and 5 exist, but
1 is not between 2 and 5
thus there might or might
not be a c with $f(c)=1$

True **False**

(b) If f is a continuous function on $[1,5]$, and differentiable on $(1,5)$, such that $f(1)=5, f(5)=1$, then by the Mean Value Theorem there is a c in $(1,5)$ with $f'(c)=-1$.

conditions of MVT are satisfied
thus there is a c in (1,5) with

$$f'(c) = \frac{f(5)-f(1)}{5-1} = \frac{1-5}{5-1} = -1$$

True **False**

(c) A function f is defined by

$$f(x) = \begin{cases} d & \text{if } x < 0 \\ \sin x + 1 & \text{if } x \geq 0 \end{cases}$$

Which constant d makes f continuous on $(-\infty, \infty)$.

$$\left.\begin{array}{l} \lim_{x \to 0^-} f(x) = d \\ \lim_{x \to 0^+} f(x) = 1 \end{array}\right\} d = 1$$

0 **1** **2** **None of these answers**

(d) Find the average value of x^2 on the interval $[0,2]$.

$$= \frac{1}{2-0} \int_0^2 x^2\,dx$$

$$= \frac{1}{2}\frac{1}{3}x^3 \Big|_0^2$$

$$= \frac{1}{6}(8-0) = \frac{8}{6} = \frac{4}{3}$$

1 **4/3** **7/3** **2**

9. (12 points) Find the equation for the tangent line at the point $(1,1)$ of the curve given implicitly by:

$$x^3 + 4xy - y^3 = 4$$

$$3x^2 + (4y + 4xy') - 3y^2 y' = 0$$

@ $(1,1)$

$$3 + (4 + 4y') - 3y' = 0$$

$$7 + y' = 0 \quad \Rightarrow y' = -7$$

slope $= -7$
point $(1,1)$ $\Bigg\}$ $\boxed{y - 1 = -7(x-1)}$ or $\boxed{y = -7x + 8}$

10. (12 points) Let $f(x) = xe^x$. Determine the intervals of increase and decrease, and the local maximum and minimum values of $f(x)$.

$$f'(x) = e^x + xe^x = (1+x)e^x$$

$f' = 0 \Rightarrow x = -1$

$f' = \emptyset$ N/A

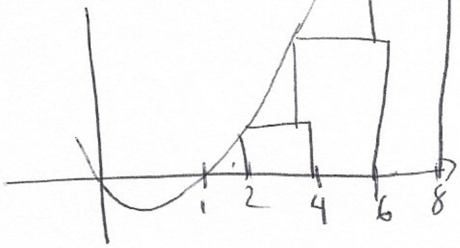
sign of f': $-$ ↘ -1 $+$ ↗

decreasing for $x < -1$ or on $(-\infty, -1)$

increasing for $x > -1$ or on $(-1, \infty)$

@ $x = -1$ absolute min

11. (10 points) Consider the area under the graph of $f(x) = x^2 - x$ over the interval $[2, 8]$. Estimate the area by dividing the interval into $n = 3$ subintervals and calculating L_3 using left endpoints.



$\Delta x = 2$

x	2	4	6
$f(x)$	2	12	30

$L_3 = (2 + 12 + 30) \cdot 2$

$= 44 \cdot 2$

$= \boxed{88}$

12. (12 points) Find the absolute maximum and minimum values of $g(x) = 3x^4 - 4x^3 - 1$ on the interval $[-2, 0]$. Show all your work!

$g'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

(i) $g' = 0$ $\boxed{x = 0}$ or $\cancel{x = 1}$ not in interval

(ii) $g' = \emptyset$ N/A

(iii) endpoints $\boxed{x = -2}$ & $\boxed{x = 0}$

compare

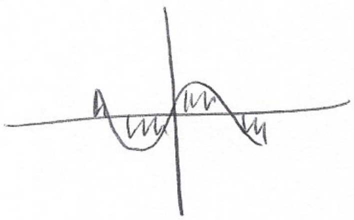
	$g(x)$
-2	$3 \cdot (-2)^4 - 4(-2)^3 - 1 = 3 \cdot 16 + 4 \cdot 8 - 1 = 79$
0	-1

min value $= -1$

max value $= 79$

13. (18 points) Suppose $\int_0^2 f(x)dx = 3$, $\int_5^2 f(x)dx = -2$, $\int_0^5 g(x)dx = 5$, and $f(x)$ is an odd function. Evaluate the following definite integrals. Your final answer should be a number!

(a) (6 points) $\int_{-2}^{2} f(x)dx = \boxed{0}$ b/c f is an odd function



(b) (6 points) $\int_{-2}^{2} (x^3 + 2x - 3)dx$

(A) x^3+2x odd $\Rightarrow \int_{-2}^{2} x^3+2x\,dx = 0$ or (B)

$= \int_{-2}^{2} -3\,dx$

$= -3x \Big|_{-2}^{2}$

$= -3\cdot(2+2)$

$= \boxed{-12}$

$= \frac{x^4}{4} + x^2 - 3x \Big|_{-2}^{2}$

$= \left(\frac{16}{4} + 4 - 6\right) - \left(\frac{16}{4} + 4 + 6\right)$

$= \boxed{-12}$

(c) (6 points) $\int_0^5 (f(x) - g(x))dx$

$\int_0^5 f(x)dx = 3 - (-2) = 5$

$= \int_0^5 f(x)\,dx - \int_0^5 g(x)dx$

$= 5 - 5$

$= \boxed{0}$

14. (10 points) Evaluate the indefinite integral:

$$\int xe^{(x^2+1)}dx$$

u-substitution

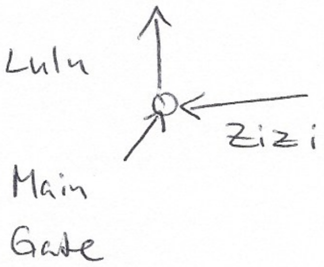
$$\boxed{\begin{aligned} u &= x^2+1 \\ du &= 2x\,dx \end{aligned}}$$

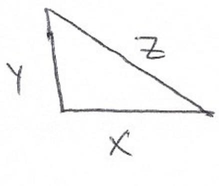
$$= \frac{1}{2}\int e^u\,du$$

$$= \frac{1}{2}e^u + c$$

$$= \boxed{\frac{1}{2}e^{x^2+1} + C}$$

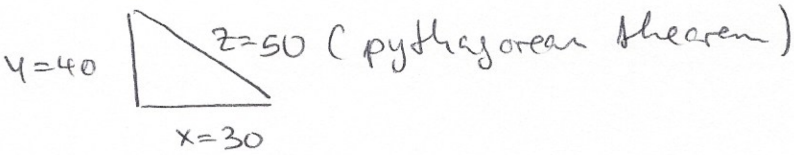
15. (12 points) Lulu, a cow, is walking at a rate of 5 feet per seconds due north away from the main gate of a range. Zizi, another cow, is walking at a rate of 10 feet per seconds due west towards the main gate. At what rate is the distance between Lulu and Zizi changing when Lulu is 40 feet north of the gate, and Zizi is 30 feet east of the gate?





Know: $\frac{dx}{dt} = -10 \qquad \frac{dy}{dt} = 5$

Want: $\frac{dz}{dt}$ when $y = 40$ & $x = 30$



$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\cancel{2}(30)(-10) + \cancel{2}\cdot(40)(5) = \cancel{2}\cdot 50\frac{dz}{dt}$$

$$-300 + 200 = 50\frac{dz}{dt}$$

$$\frac{-10\cancel{0}}{5\cancel{0}} = \frac{dz}{dt}$$

$\Rightarrow \frac{dz}{dt} = -2$ ft/s $\Rightarrow$ $\boxed{\text{distance is changing at a rate of } -2 \text{ ft/s (decreasing)}}$

16. (20 points) The function $g(x)$ is defined by $g(x) = \int_0^x f(t)dt, 0 \leq x \leq 7$. The graph of f is given below.

(a) (5 points) Calculate $g(0), g(2), g(3), g(4)$, and $g(7)$.

$g(0) = 0$

$g(2) = \frac{1}{2} \cdot 2 \cdot 2 = 2$

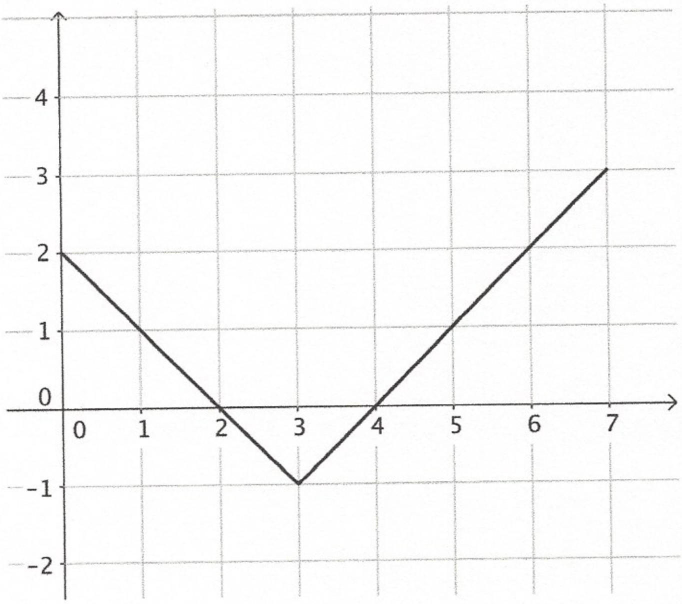
$g(3) = 2 - \frac{1}{2} = \frac{3}{2}$

$g(4) = 2 - 1 = 1$

$g(7) = 2 - 1 + \frac{1}{2} \cdot 3 \cdot 3$

$= 1 + \frac{9}{2}$

$= \frac{11}{2}$



(b) (3 points) Evaluate $g'(2)$. State the theorem(s) you are using.

FTC $g'(x) = f(x) \Rightarrow g'(2) = f(2) = 0$

(b) (3 points) On what intervals is g increasing? decreasing?

g increasing on $(0,2)$ and $(4,7)$,
decreasing on $(2,4)$.

(c) (4 points) On what intervals is g concave up? Concave down?

concave up when $g' = f$ is increasing $\Rightarrow (3,7)$
concave down when $g' = f$ is decreasing $\Rightarrow (0,3)$

(d) (5 points) Sketch a possible graph of g.