# MATH 180 Final Exam
**May 10, 2018**

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 12 pages (including this cover page) and 14 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

**TA Name:**__________________________

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded.

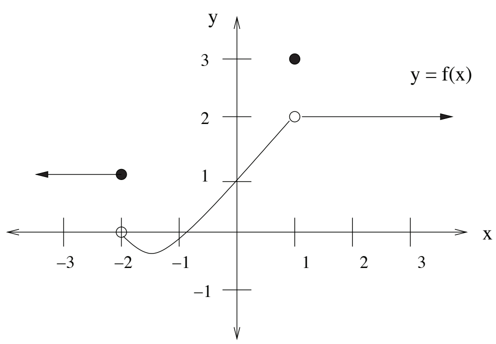
**Circle your instructor.**

- Martina Bode
- Gary (Clark) Alexander
- John Steenbergen
- Daniel Braithwaite

DO NOT WRITE ABOVE THIS LINE

1. (12 points) For each of the following questions, circle the correct answer. No explanation is necessary. No partial credit will be given.

**(I)**(8 points) Consider the function $f(x)$ whose graph is given below.




(a) $\lim_{x \to -2} f(x)$

**0** **1** **-2** **DNE**

(b) $\lim_{x \to 1} f(x)$

**1** **2** **3** **DNE**

(c) Is $f$ continuous at $x = -2$?

**Yes** **No**

(d) Is $f$ continuous at $x = 1$?

**Yes** **No**

**(II)**(4 points) Evaluate $\dfrac{d}{dx} \displaystyle\int_{\pi/8}^{2x} \tan(\theta) d\theta$

(a) $\tan(2\theta)$

(b) $\tan(2x)$

(c) $2\tan(2x)$

(d) $2\tan(2x) - 2\tan(\pi/4)$

(e) None of the above

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2. (16 points) For each of the following questions, circle the correct answer or fill in the blank. No explanation is necessary. No partial credit will be given.

**(I)** (4 points) Fill in the blank. If $f$ is differentiable everywhere, $f(1) = 3$ and $f(5) = 11$, then there is some point $c$ with $1 < c < 5$ where $f'(c) =$ ____.

**(II)** (4 points) Find the linear approximation of the function $f(x)$ at $a = 10$ given that $f(10) = 4$ and $f'(10) = 2$.

(a) $L(x) = 2 + 4(x - 10)$

(b) $L(x) = 10 + 4(x - 2)$

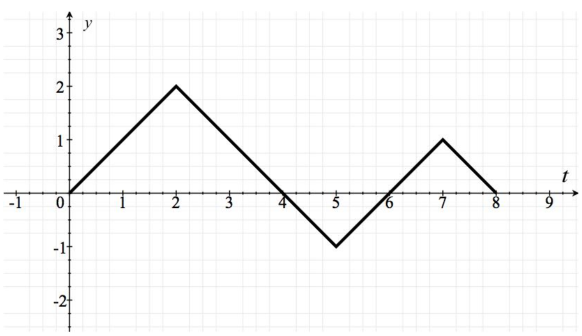
(c) $L(x) = 4 + 2(x - 10)$

(d) $L(x) = 4 + 10(x - 2)$

(e) None of the above

**(III)**(8 points)

The graph of the function $f(t)$ is given below, and define $g(x)$ as $g(x) = \int_0^x f(t)\, dt$.




(a) (4 points) Calculate $g(8)$.

**0** **4** **5** **6**

(b) (4 points) Find $g'(4)$.

**-1** **0** **1** **4**

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3. (12 points) Use the limit definition of the derivative to find $f'(x)$ for the function $f(x) = 3x^2 + 5$. No credit will be given for any other method.

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4. (26 points) Compute the following limits. If the limit does not exist, explain why it does not exist.

(a) (6 points) $\lim_{x \to 0} \dfrac{3 - 3\cos x}{x}$

(b) (6 points) $\lim_{x \to 0^-} \dfrac{1+x}{4x}$

(c) (6 points) $\lim_{x \to \infty} \dfrac{x^2 + 2x + 1}{4x^2 - 3x + 5}$

(d) (8 points) $\lim_{x \to 0} (\cos x)^{1/x}$

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5. (24 points) Differentiate the following functions, use logarithmic differentiation if needed. You do not need to simplify your answers.

   (a) (6 points) $y = \ln(1 + 2x)$

   (b) (8 points) $y = \dfrac{\sin(2x)}{x}$

   (c) (10 points) $y = x^{\tan x}$

6. (12 points) Find the slope of the tangent line to the curve $xy + y^3 = 10$ at the point $(1, 2)$.

7. (12 points) Find the maximum and minimum values of the function $f(x) = x^2 - 4x + 2$ on the interval $[-3, 3]$.

8. (12 points) A ladder 5 ft long is leaning against a vertical wall. If the base of the ladder is being pushed towards the wall at the rate of 1 ft per second, how fast will the top of the ladder move up the wall when the upper end of the ladder is 4 ft from the ground?

9. (18 points) Consider the function $f(x) = x^3 - 9x^2 - 48x + 2$.

   (a) (10 points) On what intervals is $f$ increasing? decreasing? At what values of $x$, if any, does $f$ have a local maximum? local minimum?

   (b) (8 points) On what intervals is $f$ concave upward? concave downward? At what values of $x$, if any, does $f$ have points of inflection?

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10. (12 points) Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible. Find the domain of your optimization function, and you must justify your final answer.

| | DIMENSIONS |
|---|---|
| length = | |
| width = | |

11. (18 points) Evaluate the following definite integrals.

(a) (6 points)

$$\int_{-1}^{2} x^3 dx$$

(b) (6 points)

$$\int_{-10}^{10} x^5 dx$$

(c) (6 points)

$$\int_{0}^{\pi} \sin(x) dx$$

12. (6 points) If $\mathbf{u} = \langle 1, -3 \rangle$ and $\mathbf{v} = \langle 6, 4 \rangle$, find $5\mathbf{u} - 2\mathbf{v}$.

13. (10 points) Find the projection of $\mathbf{u} = \langle 1, 8 \rangle$ onto $\mathbf{v} = \langle 4, -2 \rangle$.

14. (10 points) Suppose that a robot is pushing a box up a $45°$ incline, but the force being applied is in the horizontal direction. If the robot is applying $10N$ of force and the box moves 6 meters up the incline, how much work has been done?