

MATH 180 Final Exam

May 9, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. **YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR.** This exam contains 14 pages (including this cover page) and 14 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

Name: _____

UIN: _____

UIC Email: _____

Signature: _____

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Jenny Ross
- Mercer (Tabes) Bridges
- Matthew Lee

Write your Initials (First, Middle, Last)_____

1. (14 points) Differentiate the following functions, using logarithmic differentiation if needed. Do not simplify your answers.

(a) (4 points) $f(x) = \frac{\arcsin(x + 5)}{e^x}$

(b) (4 points) $h(x) = \tan(2^x) \ln(x)$

(c) (6 points) $g(x) = (x - 5)^{\sin(x)}$

Write your Initials (First, Middle, Last)_____

2. (14 points) Evaluate the following integrals:

(a) (4 points) $\int \left(\frac{1}{1+x^2} - \cos(x) \right) dx$

(b) (5 points) $\int_{-2}^0 (e^{3x} + 5x) dx$

(c) (5 points) $\int_{-3}^3 (\sin^3(x)) dx$

Write your Initials (First, Middle, Last)_____

3. (14 points) Compute the following limits, using L'Hôpital's Rule if needed. If a limit is $+\infty$ or $-\infty$ state which one, and if the limit does not exist, explain why.

(a) (3 points) $\lim_{x \rightarrow -2} \frac{3x^2 + 5}{x^3 - 5x^2 + 8}$

(b) (3 points) $\lim_{x \rightarrow \infty} \frac{x^2 - 5x + 4}{x^3 - 4x + 2}$

Write your Initials (First, Middle, Last)_____

Number 3 (continued)

(c) (3 points) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 6x + 9}$

(d) (5 points) $\lim_{x \rightarrow 0^+} (1 - x)^{5/x}$

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4. (10 points) Find the slope of the tangent line to $3x^2y - y^3 - x = 9$ at the point $(2, 1)$

5. (9 points) Find the absolute maximum/minimum of $f(x) = 2x^3 - 3x^2 - 12x + 5$ on the interval $[0, 3]$.

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6. (10 points) Use the **limit definition** of the derivative to find the derivative of $f(x) = 3x^2 - 1$. No other methods will receive points.

7. (9 points) Find the linear approximation to $f(x) = -3x^2 + 5x - 8$ at $x = 2$ and use it to approximate the value of $f(x)$ at $x = 2.1$. Is this an over estimation or under estimation? Justify by referencing concavity.

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8. (10 points) Let $\vec{w} = \langle -3, 4 \rangle$ and $\vec{v} = \langle -2, 1 \rangle$

(a) (2 points) Simplify $-\vec{w} + 5\vec{v}$

(b) (2 point) Find the magnitude of \vec{v}

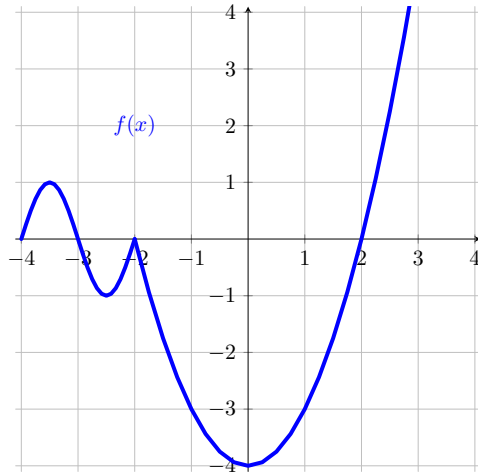
(c) (2 points) Find the value of $\vec{w} \cdot \vec{v}$

(d) (2 points) Find a unit vector parallel to \vec{w}

(e) (2 points) Find the projection of \vec{w} onto \vec{v}

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9. (7 points) For the graph of $f(x)$ below find:



(a) (2 points) State on which intervals $f'(x) < 0$ by writing your answer in interval notation, using decimals if necessary.

(b) (3 points) State on which intervals $f(x)$ is concave up/concave down by writing your answer in interval notation, using decimals if necessary.

(c) (2 points) Identify any inflection points and write your answer in a point format.

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10. (5 points) Given $A(x) = \int_{-3}^x (9 - t^2) dt$ on $[-3, 4]$, answer the following:

a. (2 point) Sketch a graph of $f(t) = 9 - t^2$.

b. (2 point) When does $A(t)$ reach its maximum value? Justify your answer.

c. (2 point) Find the following: $A(2)$.

d. (2 points) Find the following: $A'(x)$ and $A''(x)$.

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11. (4 points) (a) (2 points) Find the average value of $f(x) = -2x + 5$ on $[-2, 2]$.

(b) (2 points) Find the c on $[-2, 2]$ which satisfies the Mean Value Theorem for integrals.

12. (6 points) In an attempt to be lazy, Luke Skywalker uses the force to pull a box along the ground. Luke pulls the box from the couch at an angle of incline of $\frac{\pi}{4}$ with a force of 200 N. This force moved the box 100 meters. How much "work" is done by Luke?

Write your Initials (First, Middle, Last)_____

13. (9 points) What are the dimensions of a box, with an **open top** and square base, of maximum volume that can be constructed using 300 square feet of material.

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14. (10 points) A ladder, 13 feet in length, is leaning against a wall and starts sliding down the wall at a rate of 1 foot per second. How quickly is the base of the ladder sliding away from the wall when the bottom of the ladder is 5 feet away from the wall?

This page can be used as scratch paper. It WILL NOT BE GRADED, so please SHOW YOUR WORK WITH YOUR PROBLEMS.