MATH 180 Exam 1

February 19, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 9 pages (including this cover page) and 10 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

Name: Key

UIN:____________________

UIC Email:____________________

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Mercer (Tabes) Bridges
- Matthew Lee
- Jenny Ross

Write your Initials (First, Middle, Last)____________

1. (8 points) Suppose that $f(t)$ is the piecewise function

$$f(t) = \begin{cases} t - \tan\left(\frac{\pi t}{4}\right) & t < 3 \\ -5 & t = 3 \\ -4t^2 + 31 & t > 3 \end{cases}$$

(a) Find $\lim_{t \to 3^-} f(t)$

$$\lim_{t \to 3^-} \left(t - \tan\left(\frac{\pi t}{4}\right)\right)$$

$$= \cancel{\lim} (3) - \tan\left(\frac{(3)\pi}{4}\right)$$

$$= 3 - (-1) = \boxed{4}$$

(b) Find $\lim_{t \to 3^+} f(t)$

$$\lim_{t \to 3^+} (-4t^2 + 31)$$

$$= -4(3)^2 + 31$$

$$= -36 + 31$$

$$= -5$$

(c) Find $\lim_{t \to 3} f(t)$. Justify your answer!

$\lim_{t \to 3} f(t)$ DNE since $3 = \lim_{t \to 3^-} f(t) \neq \lim_{t \to 3^+} f(t) = 5$

(d) Is $f(t)$ continuous at $t = 3$? Why or why not?

NO. since $\lim_{t \to 3} f(t)$ DNE.

(Justifications may vary)

Write your Initials (First, Middle, Last)____________

2. (18 points) Find the derivatives of the following functions. You do not need to simplify your answers.

(a) $f(x) = \frac{7}{x^2} - 3\cos(x) + \pi$

$$f'(x) = \frac{d}{dx}(7x^{-2}) - \frac{d}{dx}(3\cos(x)) + \frac{d}{dx}(\pi)$$
$$= -14x^{-3} - 3(-\sin(x)) + 0$$
$$= \frac{-14}{x^3} + 3\sin(x)$$

(b) $g(x) = e^{-5x}\tan(3x)$

Product rule

$f(x) = e^{-5x}$ $h(x) = \tan(3x)$

$f'(x) = -5e^{-5x}$ $h'(x) = 3\sec^2(3x)$

Both require chain rule

$$g'(x) = (-5e^{-5x})(\tan(3x)) + (e^{-5x})(3\sec^2(3x))$$

(c) $h(x) = \sqrt{x^3 - 34x^2 + 10}$

Chain rule

$f(x) = \sqrt{x}$ $g(x) = x^3 - 34x^2 + 10$

$f'(x) = \frac{1}{2\sqrt{x}}$ $g'(x) = 3x^2 - 68x$

$$h'(x) = \frac{1}{2\sqrt{x^3 - 34x^2 + 10}} \cdot (3x^2 - 68x)$$

Write your Initials (First, Middle, Last)____________

3. (10 points) Let $f(x) = \dfrac{(x-1)^2}{(x+3)^2(x-1)}$.

(a) (2 points) Does $f(x)$ have any vertical asymptotes? If yes, what are they, and verify using limits.

yes. $x=-3$

$\lim_{x\to -3^-} f(x) = \lim_{x\to -3} \dfrac{(x-1)^2}{(x+3)^2(x-1)}$ "=" $\dfrac{(-4)}{0^+} = -\infty$.

(b) (2 points) Does $f(x)$ have any horizontal asymptotes? If yes, what are they, and verify using limits.

yes $y=0$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{(x-1)}{(x+3)^2} = \lim_{x\to\infty} \dfrac{(x-1)}{(x^2+6x+9)} = \lim_{x\to\infty} \dfrac{\frac{1}{x}-\frac{1}{x^2}}{1+\frac{6}{x}+\frac{9}{x^2}}$

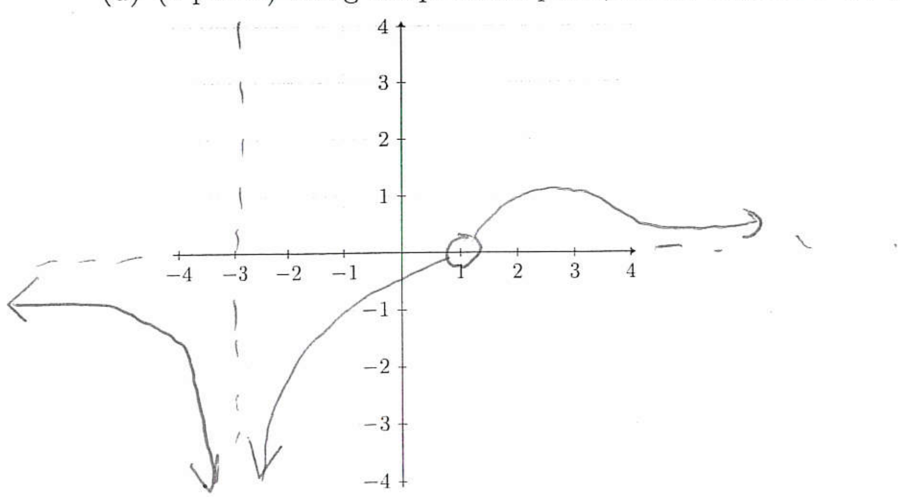
$= \dfrac{0}{1} = 0$.

(c) (2 points) Does $f(x)$ have any holes? If yes, compute the limit at the x-value.

yes. $x=1$.

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \dfrac{(x-1)}{(x+3)^2} = 0$.

(d) (4 points) Using the previous parts, sketch a graph of $f(x)$.



Write your Initials (First, Middle, Last)____________

4. (10 points) Use the **limit definition** of the derivative to compute $f'(3)$ where

$$f(x) = x^2 - 4x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 4h}{h}$$

$$= \lim_{h \to 0} 2x - 4 + h$$

$$= 2x - 4$$

$$f'(3) = 2(3) - 4 = 6 - 4 = 2.$$

Write your Initials (First, Middle, Last)____________

5. (6 points) Use the squeeze theorem to evaluate:

$$\lim_{x \to 0} \left[x^2 \sin\left(\frac{1}{x}\right) \right]$$

Remember to show all your work!

$$-1 \le \sin\left(\frac{1}{x}\right) \le 1$$

$$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$$

$$\underbrace{\lim_{x \to 0} (-x^2)}_{0} \le \lim_{x \to 0} \left(x^2 \sin\left(\frac{1}{x}\right)\right) \le \underbrace{\lim_{x \to 0} (x^2)}_{0}$$

By the squeeze Theorem

$$\lim_{x \to 0} \left(x^2 \sin\left(\frac{1}{x}\right)\right) = 0.$$

Write your Initials (First, Middle, Last)____________

6. (8 points) Consider the following table, which contains the values of the three functions, $f, g,$ and h (as well as their derivatives) at two points:

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$
2	4	1	3	-1	-3	2
3	-1	-4	1	10	5	-2

Compute the following derivatives:

(a) $\frac{d}{dx}([f(x)]^5)$ at $x = 3$.

$$\frac{d}{dx}\left([f(x)]^5\right) = 5[f(x)]^4 \cdot f'(x)$$

at $x=3$:

$$5[f(3)]^4 \cdot f'(3)$$

$$= 5(-1)^4 \cdot 10 = 50$$

(b) $\frac{d}{dx}\left(\frac{g(x)}{h(x)}\right)$ at $x = 2$.

$$\frac{d}{dx}\left(\frac{g(x)}{h(x)}\right) = \frac{g'(x)h(x) - h'(x)g(x)}{[h(x)]^2}$$

at $x=2$: $\frac{g'(2)h(2) - h'(2)g(2)}{[h(2)]^2}$

$$= \frac{(-3)(3) - (2)(1)}{(3)^2} = \frac{-11}{9}$$

Write your Initials (First, Middle, Last)____________

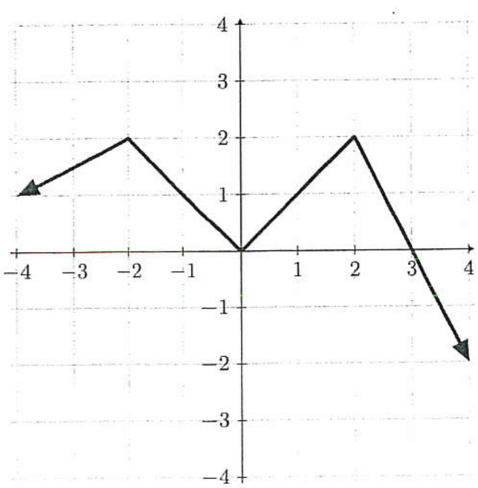
7. (6 points) Assume that a function satisfies $f(-2) = \pi$, and $f'(-2) = -6$. Find the equation of the tangent line to $y = f(x)$ at $x = -2$.

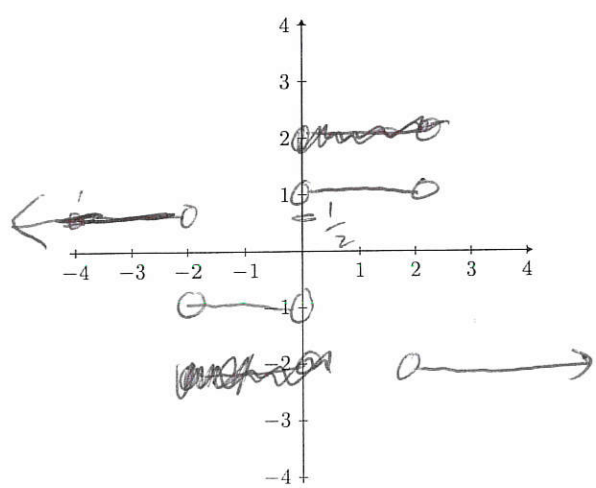
$(-2, \pi)$. $m = -6$

$y - \pi = -6(x - (-2))$

$y - \pi = -6(x+2)$

8. (8 points) The graph of the function $f(x)$ is shown below. Sketch the graph of the derivative function, $f'(x)$. Make sure you label anything that may be ambiguous in terms of their location.





Write your Initials (First, Middle, Last)____________

9. (8 points) Suppose an ant is moving along a straight path, and $s(t)$ gives its position at time t, $t > 0$. Units are in feet and seconds. Suppose that you are given that the velocity function for the ant is $s'(t) = v(t) = t^2 - 4t$.

   (a) For what value(s) of t does the ant have velocity 12 ft/s?

   $v(t) = t^2 - 4t = 12$

   $t^2 - 4t - 12 = 0$

   $(t-6)(t+2) = 0$

   $t = 6$ sec.

   (b) Find the acceleration of the ant as a function of time.

   $a(t) = v'(t) = \frac{d}{dt}(t^2 - 4t)$

   $= 2t - 4$

10. (2 points) True or ~~False~~: $f(x) = x^3 - 2x^2 - 5$ has a root in the interval $[-1, 5]$.

   $f(-1) = (-1)^3 - 2(-1)^2 - 5 < 0$

   $f(5) = (5)^3 - 2(5)^2 - 5 > 0$

This page can be used as scratch paper. It WILL NOT BE GRADED, so please SHOW YOUR WORK WITH YOUR PROBLEMS.