**Math 180** **Name (Print):** Solutions

**Exam #1** **UIN:** ____________

**9/23/2015** **Email:** ____________

**Time Limit: 2 Hours**

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This exam contains 8 pages (including this cover page) and 8 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on the top of this page.

The following rules apply:

- You may not open this exam until you are instructed to do so.
- You are expected to abide by the University's rules concerning Academic Honesty.
- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

**TA Name:** ____________

**Circle your instructor.**

- Bode
- Goldbring
- Hachtman
- Riedl
- Sinapova
- Steenbergen @ 11am
- Steenbergen @ 12pm
- Steenbergen @ 2pm

1. (6 points) Find $\lim_{x\to 0} x^4 \sin\left(\frac{1}{x^3}\right)$ using the Squeeze Theorem. Show all your work!

$-1 \le \sin\left(\frac{1}{x^3}\right) \le +1$ for all x, multiply times $x^4 \ge 0$

$-x^4 \le x^4 \cdot \sin\left(\frac{1}{x^3}\right) \le x^4$

apply the squeeze theorem

$\Rightarrow \lim_{x\to 0}(-x^4) \le \lim_{x\to 0} x^4 \cdot \sin\left(\frac{1}{x^3}\right) \le \lim(x^4)$

since $\lim_{x\to 0}(-x^4) = 0$ & $\lim_{x\to 0}(x^4) = 0$

it follows that $\lim_{x\to 0} x^4 \sin\left(\frac{1}{x^3}\right) = 0$

2. (8 points) (a) (2 points) Fill in the blanks in this statement of the Intermediate Value Theorem: Let $f$ be a function that is continuous on the interval $[a, b]$, and let $L$ be a number that is strictly between $f(a)$ and $f(b)$. Then there exists at least one number $c$ in $(a, b)$ satisfying $f(c) = L$.

(b) (6 points) Using the Intermediate Value Theorem, prove that $2x^7 - 14x^2 + 5 = 0$ has a root between 0 and 1. Remember to check that the hypotheses of the theorem are satisfied.

$f(x) = 2x^7 - 14x^2 + 5$ is continuous on $[0,1]$.

$f(0) = 5$ $f(1) = -7$

since 0 is between $f(0)$ & $f(1)$ by the Intermediate Value Theorem there is a $c$ in $(0,1)$ with $f(c) = 0$

3. (10 points) Let

$$f(x) = \frac{x^2 - 7x + 12}{(x-3)(x+2)(x-5)} = \frac{(x-3)(x-4)}{(x-3)(x+2)(x-5)}$$

(a) (4 points) Evaluate the following limits:

1. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x-4}{(x+2)(x-5)} = \lim_{x\to\infty} \frac{x-4}{x^2-3x-10} = \lim_{x\to\infty} \frac{\frac{1}{x} - \frac{4}{x^2}}{1 - \frac{3}{x} - \frac{10}{x^2}} = 0$

2. $\lim_{x\to-\infty} f(x) = 0$ (Same argument)

Based on (1.) and (2.) does $f$ have a horizontal asymptote? If yes, state the horizontal asymptote.

$y = 0$

(b) (6 points) Find all vertical asymptotes and describe the limits from the left and the right at each of the vertical asymptotes.

@ $x = -2$ Vertical Asymptote because

$\lim_{x\to-2^-} \frac{x-4}{(x+2)(x-5)} = -\infty$ of type $\frac{-6}{0^+}$

$\lim_{x\to-2^+} \frac{x-4}{(x+2)(x-5)} = +\infty$ of type $\frac{-6}{0^-}$

@ $x = 5$ Vertical Asymptote because

$\lim_{x\to5^-} \frac{x-4}{(x+2)(x-5)} = -\infty$ of type $\frac{1}{0^-}$

$\lim_{x\to5^+} \frac{x-4}{(x+2)(x-5)} = +\infty$ of type $\frac{1}{0^+}$

4. (14 points) Calculate the following limits:

(a) (6 points)

$$\lim_{x\to 9} \frac{(\sin(x-9)+2^{x-7})}{(x^2+1)}.$$

$$= \frac{\sin(0)+2^2}{9^2+1}$$

$$= \frac{4}{82}$$

$$= \boxed{\frac{2}{41}}$$

(b) (8 points)

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{(x-9)}$$

$$= \lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$$

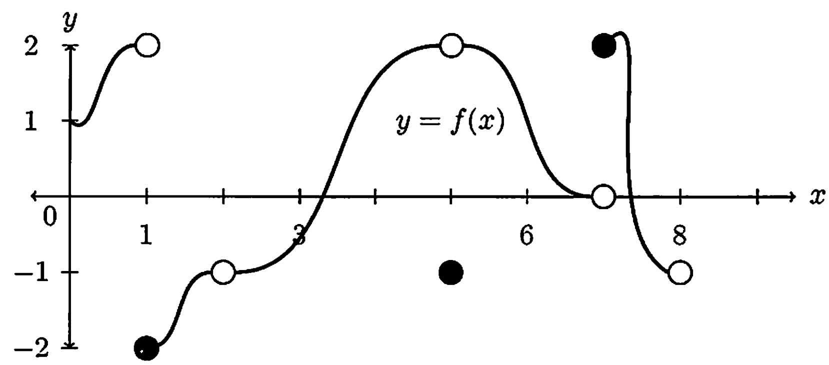
$$= \lim_{x\to 9} \frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}$$

$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \boxed{\frac{1}{6}}$$

5. (12 points) Consider the graph of the following function.




(1.) (10 points) Compute the following, or say that it does not exist:

(a) (1 point) $\lim_{x\to 1^+} f(x)$ = −2

(b) (2 points) $\lim_{x\to 5} f(x)$ = 2

(c) (2 points) $\lim_{x\to 7} f(x)$ DNE

(d) (1 point) $\lim_{x\to 8^-} f(x)$ = −1

(e) (1 point) $f(1)$ = −2

(f) (1 point) $f(2)$ DNE

(g) (1 point) $f(5)$ = −1

(h) (1 point) $f'(5)$ DNE

(2.) (2 points) Is $f(x)$ continuous at $x = 2$?

No because f(2) DNE

6. (12 points) Let $f(x)$ be a function.

   (a) (2 points) State the definition of the derivative of $f(x)$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   (b) (10 points) Now suppose $f(x) = 1 - x^2$. **Using the definition of the derivative**, find $f'(x)$ (no points will be given if the definition is not used).

$$f'(x) = \lim_{h \to 0} \frac{(\cancel{1} - (x+h)^2) - (\cancel{1} - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} - (\cancel{x^2} + 2xh + h^2)}{h}$$

$$= \lim_{h \to 0} \frac{-2xh - h^2}{h}$$

$$= \lim_{h \to 0} -2x - h$$

$$= \boxed{-2x}$$

7. (26 points) Compute the following derivatives, you do not need to simplify!

(a) (6 points) $\frac{d}{dx}\left(3x^4 - 5x + e\right)$

$= \boxed{12x^3 - 5}$

(b) (10 points) $\frac{d}{dx}\left(3x^{-3}e^x\right)$ product rule

$= (-9x^{-4})\,e^x + 3x^{-3}\cdot e^x$

$= \boxed{\left(-\frac{9}{x^4} + \frac{3}{x^3}\right)e^x}$

(c) (10 points) $\frac{d}{dx}\left(\frac{x^2}{2\cos x}\right)$ quotient rule

$$= \frac{2x \cdot 2\cos x - x^2 \cdot (-2\sin x)}{(2\cos x)^2}$$

$$= \boxed{\frac{4x\cos x + 2x^2\sin x}{(2\cos x)^2}}$$

8. (12 points) Suppose that the function $f(x)$ has an unknown formula, but its **derivative** is known and given by

$$\frac{df}{dx} = \frac{(x+1)(x-4)}{(x+2)(x-3)}$$

(a) (2 points) What is the **slope** of the unknown function $f(x)$ at $x = 0$?

$$\text{slope} = f'(0) = \frac{1 \cdot (-4)}{2 \cdot (-3)} = \frac{-4}{-6} = \boxed{\frac{2}{3}}$$

(b) (2 points) If $g(x) = f(x) + 2x$, what is $g'(0)$?

$$g'(0) = f'(0) + 2$$
$$= \frac{2}{3} + 2 = \boxed{\frac{8}{3}}$$

(c) (2 points) Find all values for $x$ where the tangent line to the unknown function $f(x)$ is **horizontal**.

i.e. slope $= 0 \Rightarrow f'(x) = 0$

$\Rightarrow \boxed{x = -1}$ & $\boxed{x = 4}$

(d) (2 points) Find all values for $x$ where the unknown function $f(x)$ is **not** differentiable.

$f'$ is undefined for

$\boxed{x = -2}$ & $\boxed{x = 3}$

(e) (4 points) Write down the equation for the **tangent line** to the unknown function $f(x)$ at $x = 2$, assuming that we know $f(2) = 10$.

point $(2, 10)$

$$\text{slope} = f'(2) = \frac{3 \cdot (-2)}{4 \cdot (-1)} = \frac{-6}{-4} = \frac{3}{2}$$

$$\Rightarrow \boxed{y - 10 = \frac{3}{2}(x - 2)}$$