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MATH 180 Exam 1
September 27, 2016

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: ANSWER KEY

University Email: ______

UIN: ______

Circle your instructor's name: Boester Dumas Embers Riedl Shulman

TA's Name: ______

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. (18 points) Find the following derivatives. DO NOT SIMPLIFY YOUR ANSWERS.

(a) $\frac{d}{dx}(e^x \sin x)$ product rule $u = e^x$ $v = \sin x$

$u' = e^x$ $v' = \cos x$

$= e^x \cdot \sin x + e^x \cdot \cos x$

(b) $\frac{d}{dx}\left(\frac{\tan x}{1+x^2}\right)$ quotient rule $u = \tan x$ $v = 1+x^2$

$u' = \sec^2 x$ $v' = 2x$

$= \frac{(\sec^2 x)(1+x^2) - 2x \cdot \tan x}{(1+x^2)^2}$

(c) $\frac{d}{dx}\left(e^{x^2+7}\right)$ chain rule

$= e^{x^2+7} \cdot 2x$

2. (6 points) Suppose the height of a ball at time t is given by $h(t) = -16t^2 + 7t + 20$ where t is in seconds and $h(t)$ is in feet. At what time is the ball traveling with an instantaneous velocity of -89 feet per second?

$v(t) = -32t + 7$ velocity

$-32t + 7 = -89$

$-32t = -96 \Rightarrow t = 3$

At $\underline{t = 3 \text{ seconds}}$ the velocity $= -89$ ft/s

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3. (8 points) Suppose the DERIVATIVE of a function f is given by $f'(x) = \frac{x^2+1}{x+3}$. Suppose also that $f(2) = 3$. What is the equation of the tangent line to $y = f(x)$ at $x = 2$?

$f'(2) = \frac{4+1}{2+3} = \frac{5}{5} = 1 = \text{slope}$

$f(2) = 3 \Rightarrow \text{point } (2,3)$

$y - 3 = 1\cdot(x-2)$

$\boxed{y = x+1}$

4. (10 points) Calculate the following limits.

(a) $\lim_{x\to -3}(2x^2 + 5x - 4)$

$= 2\cdot(-3)^2 + 5\cdot(-3) - 4 = -1$

(b) $\lim_{x\to 2} \frac{\sqrt{2}-\sqrt{x}}{2-x}$ of type $\frac{0}{0}$

$= \lim_{x\to 2} \frac{\sqrt{2}-\sqrt{x}}{2-x} \cdot \frac{\sqrt{2}+\sqrt{x}}{\sqrt{2}+\sqrt{x}}$

$= \lim_{x\to 2} \frac{\cancel{2-x}}{\cancel{2-x}} \cdot \frac{1}{\sqrt{2}+x} = \frac{1}{\sqrt{2}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$

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5. (10 points) For the following limit, write either its numerical value, $+\infty$, $-\infty$, or D.N.E. Make sure to show all your work.

$$\lim_{x \to -\infty} \frac{2x^3 + x^2 - 7}{5x^2 - 3x + 6}$$

divide by x^2

$$= \lim_{x \to -\infty} \frac{\overset{\to -\infty}{2x} + 1 - \overset{\to 0}{\frac{7}{x^2}}}{5 - \underset{\to 0}{\frac{3}{x}} + \underset{\to 0}{\frac{6}{x^2}}} = -\infty$$

6. (8 points) Find a value for q so that the limit

$$\lim_{x \to -1} \frac{x^2 + 8x + q}{x + 1}$$

exists. For the value of q that you found, find the limit.

The limit is equal to a number if $(x+1)$ is a factor of $x^2 + 8x + q$.

For ex $\boxed{q = 7}$, $x^2 + 8x + 7 = (x+1)(x+7)$

and $\lim_{x \to -1} \frac{x^2 + 8x + 7}{x+1} = \lim_{x \to -1} \frac{\cancel{(x+1)}(x+7)}{\cancel{(x+1)}} = \boxed{6}$

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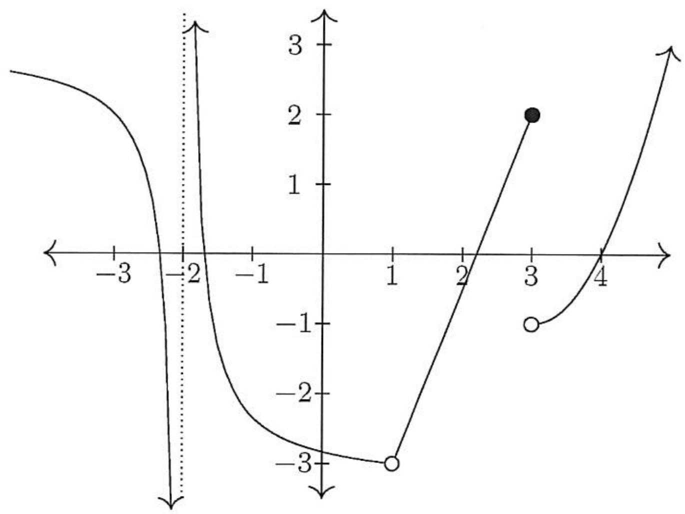
7. (10 points)



Use the graph above. For each limit below, write either its numerical value, $+\infty$, $-\infty$, or D.N.E.

(a) $\lim_{x\to -2^-} f(x)$ = $-\infty$

(b) $\lim_{x\to -2^+} f(x)$ = $+\infty$

(c) $\lim_{x\to -2} f(x)$ = D.N.E.

(d) $\lim_{x\to 1} f(x)$ = -3

(e) $\lim_{x\to 3} f(x)$ = D.N.E. because $\lim_{x\to 3} f(x) = 2 \neq \lim_{x\to 3^+} f(x) = -1$

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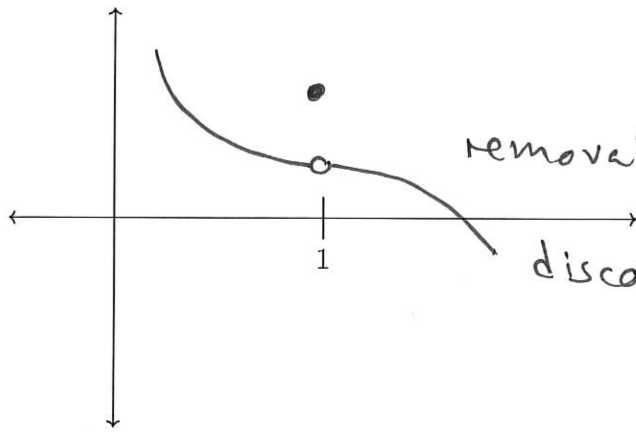
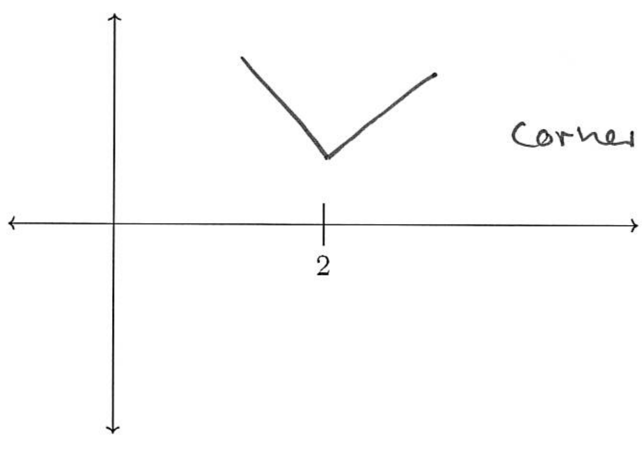
8. (10 points)

(a) On the axes below, draw a **function** that is defined at $x = 1$, has a limit as x approaches 1, but is not continuous at $x = 1$.



removable discontinuity

(b) On the axes below, draw a **function** that is continuous at $x = 2$, but is not differentiable at $x = 2$.



corner point

9. (10 points) Fill in the blanks to show that $f(x) = x^4 - 9x + 7 = 0$ has a solution.

The function $f(x) = x^4 - 9x + 7$ is a function that is __continuous__ everywhere it is defined. We wish to show that there is some value c with $f(c) = L =$ __0__. We see that L is between $f($__0__$) =$ __7__ and $f($__1__$) =$ __-1__, so by the __Intermediate Value__ Theorem, there must be some c in the interval (__0__, __1__) with $f(c) =$ __0__.

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10. (10 points) Using the **definition of the derivative**, find $f'(x)$ if $f(x) = x^2 - 5x + 6$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left((x+h)^2 - 5(x+h) + 6\right) - (x^2 - 5x + 6)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{5x} - 5h + \cancel{6} - \cancel{x^2} + \cancel{5x} - \cancel{6}}{h}$$

$$= \lim_{h\to 0} 2x + h - 5$$

$$= \boxed{2x - 5}$$

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