MATH 180 Exam 1

October 3, 2017

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. This exam contains 8 pages (including this cover page) and 8 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

TA Name:____________________ ANSWER KEY

First name (please write as legibly as possible within the boxes)

Last name

Student NetID

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Martina Bode
- Jenny Ross
- Drew Shulman
- Matthew Woolf
- Sherwood Hachtman

DO NOT WRITE ABOVE THIS LINE!!

1. (10 points) Use the limit definition to compute the derivative function $f'(x)$ for the function:

$$f(x) = \sqrt{6x+1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{6(x+h)+1} - \sqrt{6x+1}}{h} \quad \text{of type } \frac{0}{0}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{6x+6h+1} - \sqrt{6x+1}\right)\left(\sqrt{6x+6h+1} + \sqrt{6x+1}\right)}{h \cdot \left(\sqrt{6x+6h+1} + \sqrt{6x+1}\right)}$$

$$= \lim_{h \to 0} \frac{(\cancel{6x} + 6h + \cancel{1}) - (\cancel{6x} + \cancel{1})}{h \cdot \left(\sqrt{6x+6h+1} + \sqrt{6x+1}\right)}$$

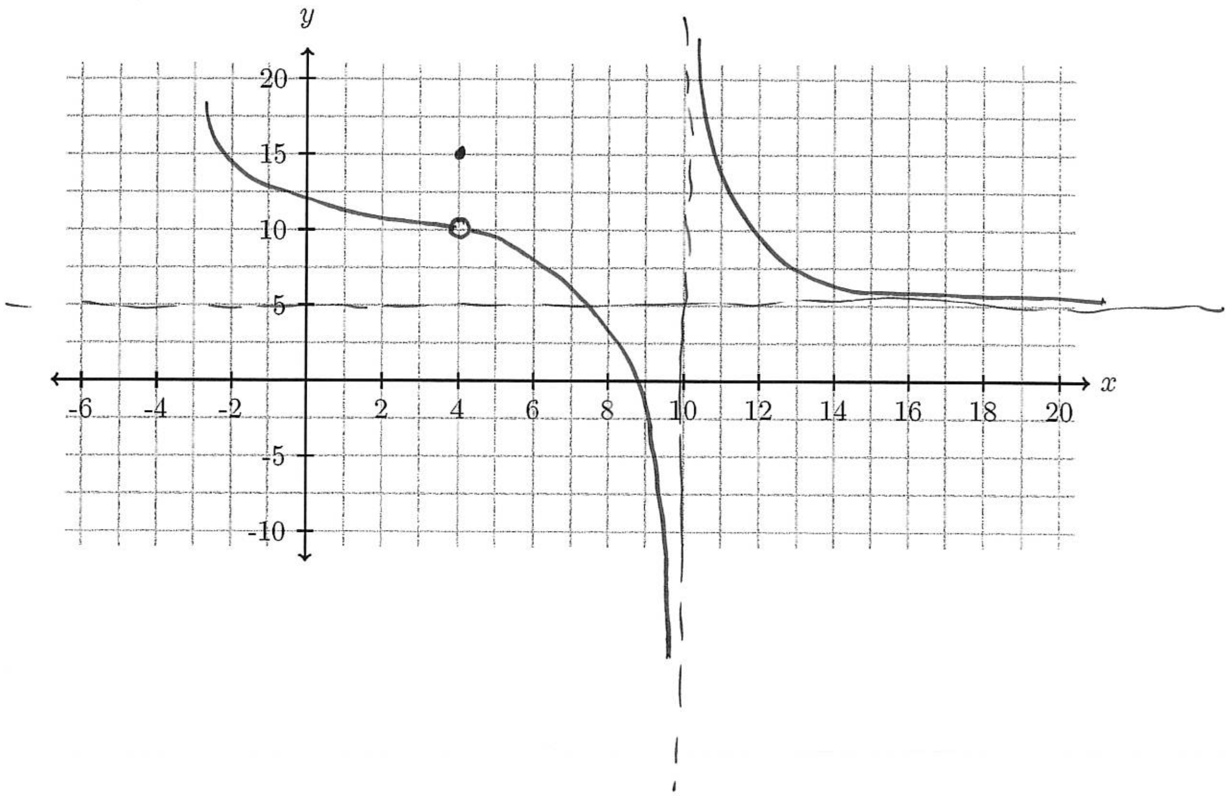
$$= \lim_{h \to 0} \frac{6\cancel{h}}{\cancel{h} \cdot \left(\sqrt{6x+6h+1} + \sqrt{6x+1}\right)}$$

$$= \frac{6}{\sqrt{6x+1} + \sqrt{6x+1}}$$

$$= \frac{6}{2\sqrt{6x+1}} = \boxed{\frac{3}{\sqrt{6x+1}}}$$

2. (12 points) Draw a graph of a function $f(x)$ that satisfies all of the following properties:

(a) $f(4) = 15$
(b) $\lim_{x\to 4} f(x) = 10$ } removable discontinuity @ x=4
(c) $\lim_{x\to 10^-} f(x) = -\infty$
(d) $\lim_{x\to 10^+} f(x) = \infty$ } x=10 vertical asymptote
(e) $\lim_{x\to\infty} f(x) = 5$ y=5 Horizontal asymptote
(f) $\lim_{x\to-\infty} f(x) = \infty$ ← on the left graph takes off to + ∞



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3. (24 points) Evaluate the following limits. If the limit is infinite, state whether it is ∞ or $-\infty$. Clearly explain your reasoning, stating theorems as needed.

(a) (6 points) $\lim_{z\to\pi} \frac{2+\cos(z)}{z^2+1}$ $= \frac{2+\cos(\pi)}{\pi^2+1} = \frac{2-1}{\pi^2+1} = \boxed{\frac{1}{\pi^2+1}}$

(b) (6 points) $\lim_{x\to 3} \frac{x^2-10x+21}{x-3}$ of type $\frac{0}{0}$, factor numerator

$= \lim_{x\to 3} \frac{(x-\cancel{3})(x-7)}{x-\cancel{3}}$

$= \boxed{-4}$

(c) (6 points) $\lim_{x\to\infty} \frac{3x^2+x+5}{1+5x+7x^2}$ of type $\frac{\infty}{\infty}$, divide by x^2

$= \lim_{x\to\infty} \frac{3+\overset{\to 0}{\frac{1}{x}}+\overset{\to 0}{\frac{5}{x^2}}}{\underset{\to 0}{\frac{1}{x^2}}+\underset{\to 0}{\frac{5}{x}}+7} = \boxed{\frac{3}{7}}$

(d) (6 points) $\lim_{x\to 2^-} \frac{x^2}{x-2}$ $= \boxed{-\infty}$ of type $\frac{4}{0^-}$

$x \to 2^- \Rightarrow x < 2 \Rightarrow x-2 < 0$

4. (10 points) A function f is defined by

$$f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 6 & \text{if } x = 0 \\ \sin x + 1 & \text{if } x > 0 \end{cases}$$

(a) (3 points) Find $\lim_{x\to 0^-} f(x)$.

$$= \lim_{x\to 0} \cos x = \cos(0) = 1$$

(b) (3 points) Find $\lim_{x\to 0^+} f(x)$.

$$= \lim_{x\to 0} (\sin x + 1) = 0 + 1 = 1$$

(c) (2 points) Find $\lim_{x\to 0} f(x) = 1$

(d) (2 points) Is f continuous at $x = 0$? No, because $f(0) = 6 \neq \lim_{x\to 0} f(x) = 1$

5. (10 points) Evaluate $\lim_{x\to 0} x^2 \sin(1/x)$. Clearly explain your reasoning, stating theorems as needed.

$-1 \leq \sin(\frac{1}{x}) \leq +1$ true for all $x \neq 0$

since $x^2 > 0$ for $x \neq 0$ we can multiply this inequality by x^2.

$$-x^2 \leq x^2 \sin(\tfrac{1}{x}) \leq +x^2$$

by Squeeze Theorem

$$0 = \lim_{x\to 0} (-x^2) \leq \lim_{x\to 0} x^2 \sin(\tfrac{1}{x}) \leq \lim_{x\to 0} x^2 = 0$$

$$\Rightarrow \lim_{x\to 0} x^2 \sin(\tfrac{1}{x}) = 0$$

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6. (20 points) Find the derivatives of the following functions. Do not simplify your answers.

(a) (5 points) $f(t) = 9t - \frac{5}{t^6} + 3^4$ $= 9t - 5t^{-6} + 3^4$ power rule

$f'(t) = 9 + \frac{30}{t^7}$

(b) (5 points) $f(x) = \sin(7x^3 + 6x)$ chain rule

$f'(x) = \cos(7x^3+6x)(21x^2+6)$

(c) (5 points) $f(x) = \frac{\tan(3x)}{\sqrt{x}}$ quotient $u = \tan(3x)$ $v = \sqrt{x}$

$u' = 3\sec^2(3x)$ $v' = \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{3\sec^2(3x)\cdot\sqrt{x} - \frac{\tan(3x)}{2\sqrt{x}}}{(\sqrt{x})^2}$

(d) (5 points) Find $h'(2)$ given that $h(x) = x^2(g(x) + 2)$ with $g(2) = 3$, and $g'(2) = 5$.

product rule $u = x^2$ $v = g(x)+2$

$u' = 2x$ $v' = g'(x)$

$h'(x) = 2x(g(x)+2) + x^2 \cdot g'(x)$

$h'(2) = 2\cdot 2(g(2)+2) + 2^2\cdot g'(2)$

$= 4\ (3+2) + 4\cdot 5$

$= 20 + 20$

$= \boxed{40}$

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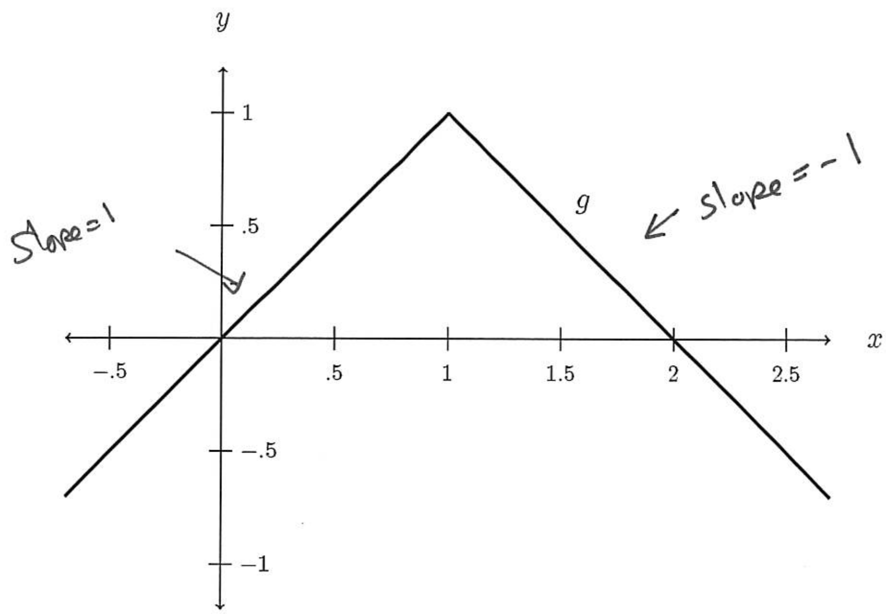
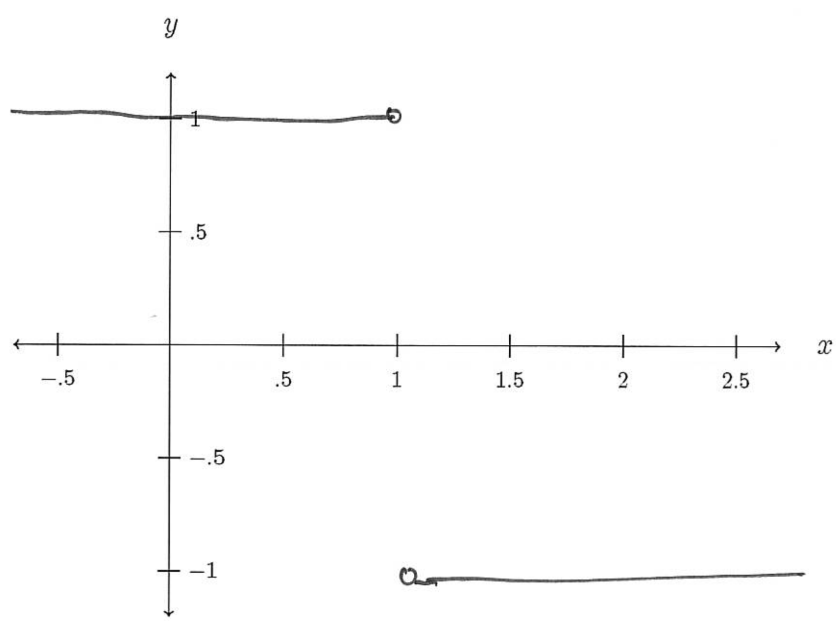
7. (6 points) The graph of $g(x)$ is shown below.



Sketch the graph of the derivative $g'(x)$.



8. (8 points) Fill in the blanks in the left column to complete the statements to show that $x^3 + 2x = 1$ has a solution.

Choose your answers from the items in the column to the right.

Suppose $f(x) =$ x^3+2x
(or x^3+2x+1)

is a continuous function

on the interval $[0,1]$.

Let $L =$ 1 be a number
(or 0)

between $f(0)=0$ and $f(1)=3$.
(or $f(0)=-1$ and $f(1)=2$)

Then by the Intermediate Value Theorem

there is a c in the interval $(0,1)$

such that:

$f(c)=1$
(or $f(c)=0$)

- $f(x) = x^3 + 2x$
- $f(x) = x^3 + 2x - 1$
- $[0, 1]$
- $[1, 3]$
- $(0, 1)$
- $(1, 3)$
- 0
- 1
- 2
- 3
- $f(0) = 0$
- $f(0) = -1$
- $f(1) = 3$
- $f(1) = 2$
- Squeeze Theorem
- Intermediate Value Theorem
- Fundamental Theorem
- $(0, 1)$
- $(1, 3)$
- $(0, 3)$
- $(-1, 2)$
- $f(c) = 0$
- $f(c) = 1$
- $f(c) = 2$
- $f(c) = 3$

Note: This problem has 2 possible set ups, one with $f(x)=x^3+2x$, the other with $f(x)=x^3+2x+1$.