MATH 180 Exam 1

October 1, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 8 pages (including this cover page) and 9 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty. Please put your initials on each page.

Your Name:__________________________

Your NetID:__________________________

TA Name:__________________________

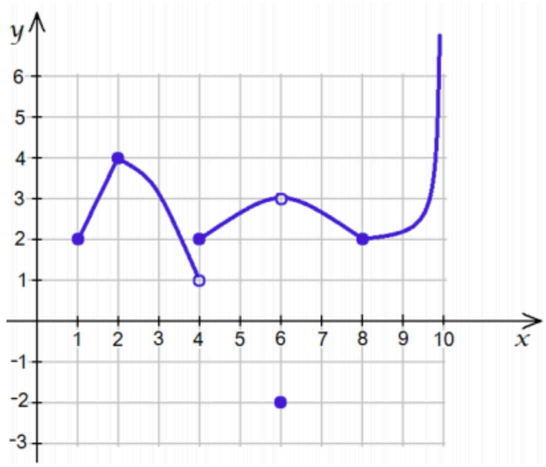
Circle your instructor.

- Martina Bode
- Shavila Devi
- Vi Diep
- Matthew Lee
- John Steenbergen

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

1. (12 points) (1 point each) Determine each of the following based on the picture of the function $f(x)$ shown below.



(a) $\lim_{x\to 1^+} f(x)$

(b) $\lim_{x\to 2} f(x)$

(c) $f(2)$

(d) Is the function continuous at $x = 2$?

(e) $\lim_{x\to 4} f(x)$

(f) $f(4)$

(g) Is the function continuous at $x = 4$?

(h) $\lim_{x\to 6} f(x)$

(i) $f(6)$

(j) Is the function continuous at $x = 6$?

(k) $\lim_{x\to 8} f(x)$

(l) $\lim_{x\to 10^-} f(x)$

2. (15 points) Evaluate the following limits. If the limit is infinite, state whether it is ∞ or $-\infty$. Justify your answers.

   (a) (5 points) $\displaystyle\lim_{x\to 3} \frac{x^2 + x - 12}{3x - 9}$

   (b) (5 points) $\displaystyle\lim_{x\to 8} \frac{\sqrt{x+1} - 3}{x - 8}$

   (c) (5 points) $\displaystyle\lim_{x\to 4^-} \frac{x - 10}{x - 4}$

3. (10 points) Use the **limit definition** of the derivative to compute $f'(x)$ where $f(x) = 3x^2 + 4$. No other methods will earn points. Show all your work!

4. (6 points) Show that $\lim_{x\to 0}\left[x^6\cos\left(\frac{1}{x^6}\right)\right] = 0$ by filling in the blanks below.

For all $x \neq 0$, we know that: $\_\_\_\_\_\_ \leq \cos\left(\frac{1}{x^6}\right) \leq \_\_\_\_\_\_$

Note that $x^6 \geq 0$ for all x. Then multiplying the inequality by x^6 yields:

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x^6\cos\left(\frac{1}{x^6}\right) \leq \_\_\_\_\_\_\_\_\_\_\_\_$$

The squeeze theorem implies that:

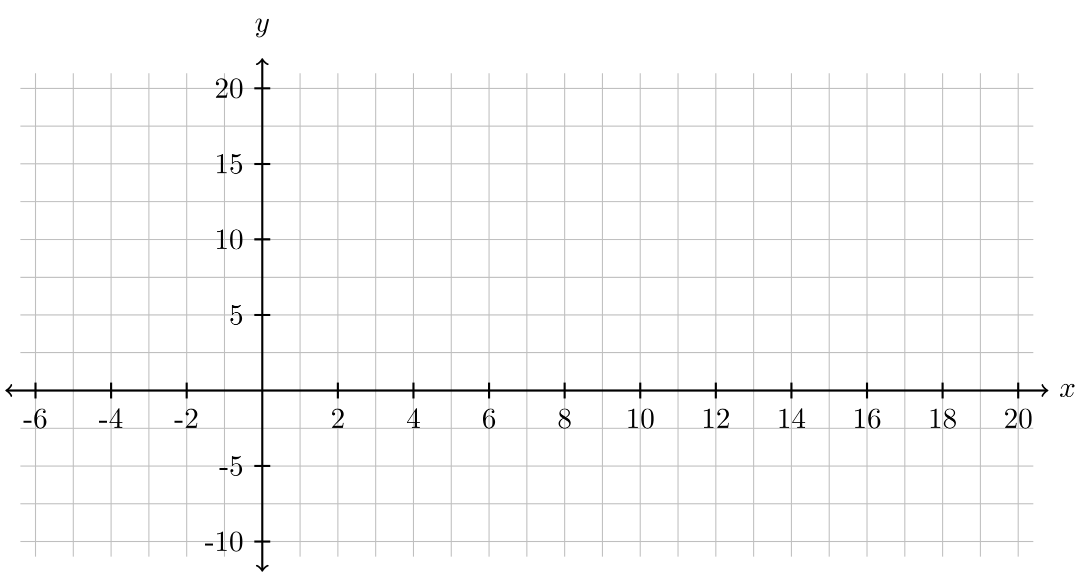
$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq \lim_{x\to 0}\left[x^6\cos\left(\frac{1}{x^6}\right)\right] \leq \_\_\_\_\_\_\_\_\_\_\_\_$$

The limits on the left and right of the inequality are both 0, we conclude that: $\lim_{x\to 0}\left[x^6\cos\left(\frac{1}{x^6}\right)\right] = 0$

5. (8 points) Suppose f is a function with the following properties:

- $\lim_{x\to 10^-} f(x) = \infty$
- $\lim_{x\to 10^+} f(x) = -\infty$
- $\lim_{x\to \infty} f(x) = 15$
- $\lim_{x\to -4^+} f(x) = \infty$

Sketch a possible graph of $y = f(x)$.



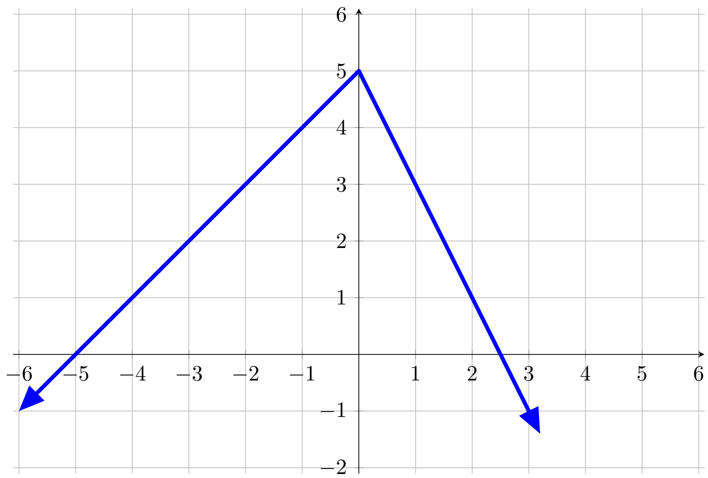
6. (15 points) Find the derivatives of the following functions. No need to simplify!

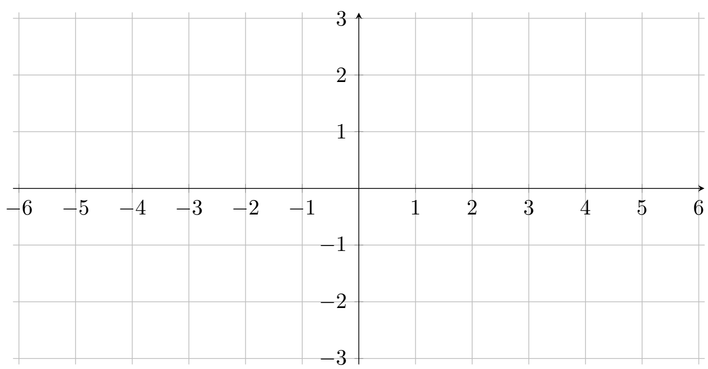
   (a) (5 points) $f(x) = e^{5x}\left(\frac{2}{x} + 4\sqrt{x}\right)$

   (b) (5 points) $g(t) = \frac{1+t^3}{5-t^2}$

   (c) (5 points) $h(\theta) = \tan(\sqrt{\theta})$

7. (8 points) For the function below sketch the derivative.





8. (10 points) An object's height (measured in feet) is defined by $s(t) = 0.5t^2 + 12$ where t is the time, measured in seconds.

   (a) (2 points) Find the height of the object at $t = 10$ seconds.

   (b) (4 points) Find the average velocity of the object between $t = 0$ and $t = 2$ seconds.

   (c) (4 points) Find the instantaneous velocity of the object at $t = 2$ seconds.

Multiple Choice

9. (16 points) Circle the correct answers.

(a) The limit $\lim_{x\to\infty} \dfrac{5x^2 + 10x + 1}{x^3 - 4}$ is equal to:

(i) 0 (ii) 5 (iii) ∞ (iv) None of these answers

(b) If f is a continuous function on $[1, 4]$ such that $f(1) = -2, f(4) = 3$, then by the Intermediate Value Theorem there is a c in $(1, 4)$ with $f(c) = 1$.

(i) True (ii) False (iii) Not enough information given

(c) If f is a differentiable function on $[1, 5]$ such that $f(3) = 5, f'(3) = 2$, then an equation of the tangent line is given by:

(i) $y = 2x - 1$ (ii) $y = 2x + 5$ (iii) $y = 2x + 10$ (iv) $y = 3x - 5$

(d) A function $h(x)$ is defined by $h(x) = (f(x))^3$.
Suppose we know that $f(1) = 2$, and $f'(1) = 10$, find $h'(1)$.

(i) 12 (ii) 120 (iii) 300 (iv) None of these answers