Math 180
Spring 2015
First Midterm
2/11/2015
Time Limit: 2 Hours

Name (Print): ______________________________

This exam contains 11 pages (including this cover page) and 10 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

The following rules apply:

- You are expected to abide by the University's rules concerning Academic Honesty.
- You may *not* use your books, notes, or any electronic device including cell phones.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You must complete your work in the space provided. No extra paper will be provided.

Circle your instructor.

- Cabrera
- Cohen
- Groves
- Kobotis
- Lowman
- Shulman
- Steenbergen

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

TA Name: ______________________________

1. (10 points) Using the definition of the derivative as the limit of a difference quotient, find $\frac{dy}{dx}$ if $y = \frac{x^2}{2} - 5$.

SOLUTION:

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h\to 0} \frac{\left[\frac{(x+h)^2}{2} - 5\right] - \left[\frac{x^2}{2} - 5\right]}{h} = \lim_{h\to 0} \frac{x^2 + 2x + h^2 - x^2}{2h} \\ &= \lim_{h\to 0} \frac{h(2x+h)}{2h} = \lim_{h\to 0} \frac{2x+h}{2} = x. \end{aligned}$$

GRADING RUBRIC:

0 points – If the student does not use the definition of the derivative

2 points – If the student sets up the limit definition using $y = \frac{x^2}{2} - 5$

4 points – If the student simplifies by expanding out $(x+h)^2$ and cancels the 5's

6 points – If the student cancels out the x^2's and arrives at $\frac{2xh + h^2}{2h}$

8 points – If the student cancels the h's

10 points – Correct calculation of the limit to find $\frac{dy}{dx} = x$

-2 points – If a student makes a small algebra error

-2 points – If the student has incorrect labelling of solution (i.e. missing a limit, taking the limit of nothing (i.e. such as $\lim_{h\to 0} = x$), having a limit in the final answer)

2 points (maximum) – If the student does not mention a limit

2. (10 points) Calculate the following limit by finding its value or justify why it does not exist.
$\lim_{x\to 0} \frac{\sin(4x)}{2x}$

SOLUTION:

$$\lim_{x\to 0} \frac{\sin(4x)}{2x} = \frac{4}{2} \underbrace{\lim_{x\to 0} \frac{\sin(4x)}{4x}}_{=1} = \frac{4}{2} = 2$$

GRADING RUBRIC:

0 points – If the student made an algebraic mistake with $\sin(4x)$

1 point – If the student knew $\sin(x)/x \to 1$ as $x \to 0$

3 points (maximum) – If the student had the correct answer with $\frac{\sin(4x)}{4x}$ but with an algebraic mistake

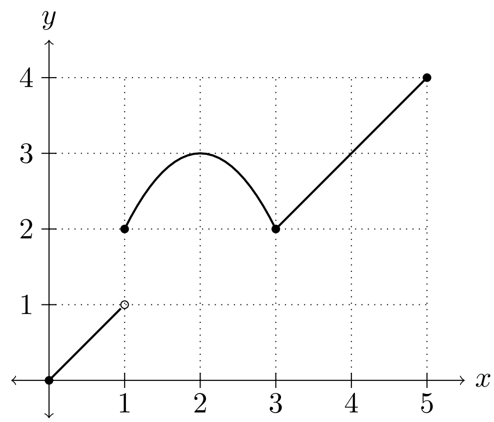
-2 points – If the student correctly used L'Hopital's rule without mentioning it

-2 points – If the student prematurely used L'Hopital's rule

-2 points – If the student introduced $u = 4x$, but wrote $\lim_{x\to 0} \frac{\sin u}{u} = 1$

-2 points – If the student substituted 0 but did not take away the limit

3. (10 points) The graph of $f(x)$ is below. Find all x-values such that $0 < x < 5$ where $f'(x) = 0$ or where $f'(x)$ does not exist. Justify your x-values.



SOLUTION: $f'(x) = 0$ when $x = 2$ since there is a horizontal tangent line at this point. $f'(x)$ does not exist at $x = 1$ since f is discontinuous at this point; $f'(x)$ also does not exist at $x = 3$ since there is a cusp at this point.

GRADING RUBRIC:

3 points (each) – If the student found a point where $f'(x) = 0$ or $f'(x)$ does not exist and said which it is WITH PROPER JUSTIFICATION

1 point – If the student got all three points with proper justification

4. (10 points) Determine the end behavior of $f(x) = e^x + \dfrac{1}{x}$ as x approaches $\pm\infty$.

SOLUTION: We know that $\displaystyle\lim_{x\to\pm\infty} \frac{1}{x} = 0$ so

$$\begin{aligned} \lim_{x\to+\infty} \left(e^x + \frac{1}{x}\right) &= \lim_{x\to+\infty} e^x = +\infty \\ \lim_{x\to-\infty} \left(e^x + \frac{1}{x}\right) &= \lim_{x\to-\infty} e^x = 0 \end{aligned}$$

Therefore the end behavior to the right is that f grows without bound towards $+\infty$ and the end behavior to the left is that f approaches 0, so $y = 0$ is a horizontal asymptote.

GRADING RUBRIC: For the end behavior to the right:

2 points – If the student wrote that $\displaystyle\lim_{x\to+\infty} \frac{1}{x} = 0$

2 points – If the student wrote that $\displaystyle\lim_{x\to+\infty} e^x = +\infty$

5 points – If the student calculates $\displaystyle\lim_{x\to+\infty} f(x) = +\infty$

For the end behavior to the left:

2 points – If the student wrote that $\displaystyle\lim_{x\to-\infty} \frac{1}{x} = 0$

2 points – If the student wrote that $\displaystyle\lim_{x\to-\infty} e^x = 0$

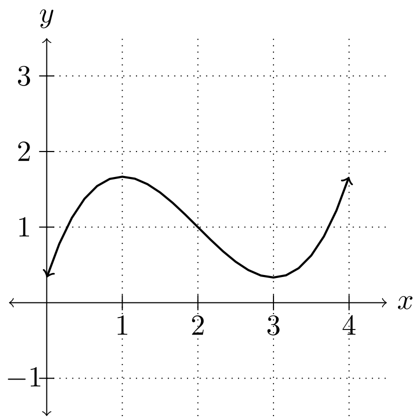
5 points – If the student calculates $\displaystyle\lim_{x\to-\infty} f(x) = 0$

-1 point – If the student jumped from $\displaystyle\lim_{x\to+\infty} \left(e^x + \frac{1}{x}\right)$ to $+\infty$

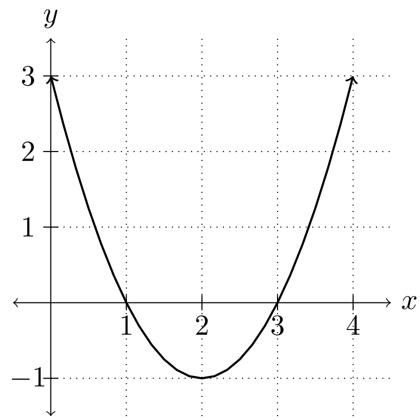
-1 point – If the student jumped from $\displaystyle\lim_{x\to-\infty} \left(e^x + \frac{1}{x}\right)$ to 0

-1 point – If the student's final answer is "$\displaystyle\lim_{x\to+\infty} f(x)$ does not exist"

5. (10 points) The graphs of f and f' are below. Determine which graph is f and which graph is f'. You need to give three reasons why you made your choice.



Graph 1



Graph 2

SOLUTION: Graph 1 is f and Graph 2 is f'. Some justifications why are:

- Graph 1 has a horizontal tangent line at $x = 1$ and Graph 2 has a zero at $x = 1$
- Graph 1 has a horizontal tangent line at $x = 3$ and Graph 2 has a zero at $x = 3$
- Graph 1 has a negative tangent line slope at $x = 2$ and Graph 2 is negative at $x = 2$
- If Graph 2 were to be f and Graph 1 were to be f' then at $x = 2$ Graph 2 should have a zero, but it does not

GRADING RUBRIC:

0 points – If the student did not write which is f and which is f'

1 point – If the student states that Graph 1 is f and Graph 2 is f'

3 points (each) – If the student gave a reason why s/he made their decision

Note: If a student said "Graph 1 is a cubic and Graph 2 is a quadratic, so ..." then this justification did not count since the graphs may not be polynomials. Also, if a student gave more than three justifications, then only the first three listed (in order) were graded.

6. (10 points) Differentiate the following function. Do not simplify your answer. $\dfrac{\sin x}{3x^2 + \tan x}$

SOLUTION:

$$\frac{(3x^2 + \tan x)\cos x - \sin x(6x + \sec^2 x)}{(3x^2 + \tan x)^2}$$

GRADING RUBRIC: If an attempt is made to use the quotient rule, then:

3 points – If the student found the derivative of the numerator OR denominator but not both

6 points – If the student found the derivative of the numerator AND denominator

10 points – If the student has the correct answer

-2 points (per term) – If the student is missing parentheses in one of the terms

-2 points – If the student used incorrect notation, such as $f(x) = f'(x)$

-4 points – If the student calculated a trigonometric derivative incorrectly

If no attempt was made to use the quotient rule (note that f'/g' does not qualify as an attempt), then:

0 points – If the student did not attempt the quotient rule

7. (10 points) Differentiate the following function. Do not simplify your answer. $4\sqrt{x}e^x$

SOLUTION:

$$4\sqrt{x}e^x + 4 \cdot \frac{1}{2\sqrt{x}}e^x = 4\sqrt{x}e^x + \frac{2e^x}{\sqrt{x}} = 4x^{1/2}e^x + 2e^x x^{-1/2}$$

GRADING RUBRIC: If an attempt is made to use the product rule, then:

3 points – If the student found the derivative of either $4\sqrt{x}$ OR e^x but not both

6 points – If the student found the derivative of $4\sqrt{x}$ AND e^x

10 points – If the student has the correct answer

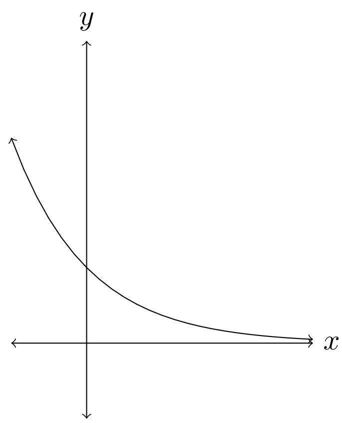
If no attempt was made to use the product rule (note that $f' \cdot g'$ does not qualify as an attempt), then:

0 points – If the student did not attempt the product rule

8. (10 points) On the axes provided, draw the graph of a function $g(x)$ with the following properties:

- g is continuous on $(-\infty, \infty)$
- $g'(0) < 0$
- $\lim_{x \to +\infty} g(x) = 0$

SOLUTION: Here is one possible answer.



GRADING RUBRIC:

3 points – If the graph depicts exactly ONE of g continuous on $(-\infty, \infty)$, $g'(0) < 0$, or $\lim_{x \to +\infty} g(x) = 0$

6 points – If the graph depicts exactly TWO of g continuous on $(-\infty, \infty)$, $g'(0) < 0$, or $\lim_{x \to +\infty} g(x) = 0$

10 points – If the graph depicts all THREE of g continuous on $(-\infty, \infty)$, $g'(0) < 0$, and $\lim_{x \to +\infty} g(x) = 0$

9. (10 points) Find all x-values where the slope of the tangent line to the curve $y = x^3 + x$ is equal to 4.

   SOLUTION: $y' = 3x^2 + 1$. We seek values where $y' = 4$, so $3x^2 + 1 = 4$. This means $3x^2 = 3$ or $x^2 = 1$ and so $x = \pm 1$.

   GRADING RUBRIC:

   4 points – If the student calculates y'

   6 points – If the student sets $y' = 4$

   10 points – If the student solves $y' = 4$

   -2 points – If the student concludes that $x^2 = 1$ means $x = 1$ and forgets $x = -1$

   -2 points – If the student commits a minor algebra error

10. (10 points) Use the Squeeze Theorem to find the following limit. $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x^3}\right)$

SOLUTION:

$$\begin{aligned} -1 \le &\quad \sin\left(\frac{1}{x^3}\right) &\le 1 \\ -x^2 \le &\quad x^2 \sin\left(\frac{1}{x^3}\right) &\le x^2 \end{aligned}$$

Since $\lim_{x\to 0} -x^2 = \lim_{x\to 0} x^2 = 0$, we know by the Squeeze Theorem that $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x^3}\right) = 0$.

GRADING RUBRIC:

0 points – If the students recognizes evaluating the function at $x = 0$ is undefined but does nothing else

3 points – If the student bounds $x^2 \sin\left(\frac{1}{x^3}\right)$ from below or above BUT NOT BOTH

6 points – If the student bounds $x^2 \sin\left(\frac{1}{x^3}\right)$ from below AND above

8 points – If the student shows the bounds have the same limit as $x \to 0$ (the student will not receive credit for this part if the limit is not present)

10 points – If the student concludes the limit using the Squeeze Theorem