MATH 180 Exam 2

October 29, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 10 pages (including this cover page) and 9 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty. Please put your initials on each page.

Your Name:__________________________

Your NetID:__________________________

TA Name:__________________________

Circle your instructor.

- Martina Bode
- Shavila Devi
- Vi Diep
- Matthew Lee
- John Steenbergen

The following rules apply:

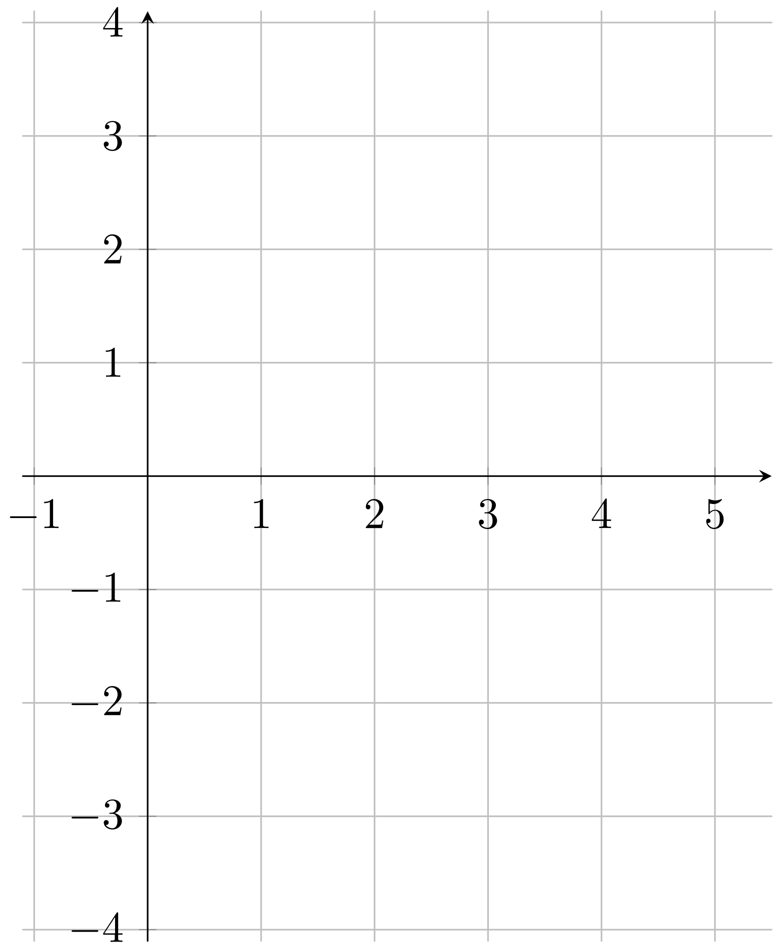
- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

1. (10 points) Find an equation of the line tangent to the curve $x^3 - xy + y^2 = 7$ at the point $(1, -2)$.

2. (10 points) Find the absolute maximum/minimum of $f(x) = x^3 - 3x$ on the interval $[0, 3]$. Also, specify where the absolute maximum/minimum occur.

3. (8 points) Sketch a graph of a function f that is continuous on $[0, 4]$ and satisfies the following conditions.

- $f(0) = 0$ and $f(4) = 0$
- $f' > 0$ on $(0, 2)$ and $f' < 0$ on $(2, 4)$
- $f'(2) = 0$
- $f'' > 0$ on $(0, 1)$ and $(3, 4)$, and $f'' < 0$ on $(1, 3)$



4. (14 points) Consider the function $f(x) = x^3 - 9x^2 + 24x$. We have done some of the calculations for you, you may use that:

$$f(0) = 0, f(1) = 16, f(2) = 20, f(3) = 18, f(4) = 4$$

(a) (4 points) On which intervals is the function $f(x)$ increasing? decreasing?

(b) (2 points) Does $f(x)$ have any local maxima or local minima? State your answer(s) in point form.

(c) (4 points) On which intervals is f concave upward? concave downward? Does f have any inflection point(s)? If so, justify and state your answer in point form.

(d) (4 points) Use your answers from above to sketch the graph of $f(x)$, make sure to label relevant points.

5. (6 points) At Noon, a car is traveling at 30 mph. Two hours later, the car is traveling at 50 mph. Show, by filling in the blanks below, that at some time between Noon and 2pm the car's acceleration is exactly 10 miles per hours squared.

   Define the function $g(t)$ to be the speed of the car at time t, where t is measured in hours after Noon.

   Then the function g is __________ on the interval $[0, 2]$, and differentiable on the interval ______ .

   By the Mean Value theorem there is a time c, for c in the interval ______ , such that

   $$g'(c) = \rule{4cm}{0.4pt}$$

   Since $g'(c)$ represents the acceleration of the car, it follows that the acceleration at time c

   is exactly equal to ______ .

6. (8 points) This problem is a Multiple Choice problem. Circle the correct answers.

   (I) Find all local max and min for the function f where $f'(x) = (x - 2)^2(x - 3)$.

   A. Local max at $x = 2$, local max at $x = 3$
   B. Local max at $x = 2$, local min at $x = 3$
   C. Local min at $x = 2$, local max at $x = 3$
   D. Local max at $x = 2$, local min at $x = 3$
   E. Neither local max nor local min at $x = 2$, local min at $x = 3$
   F. Neither local max nor local min at $x = 2$, local max at $x = 3$

   (II) Consider a function f where $f'(2) = 0$ and $f''(2) = 1$. Then:

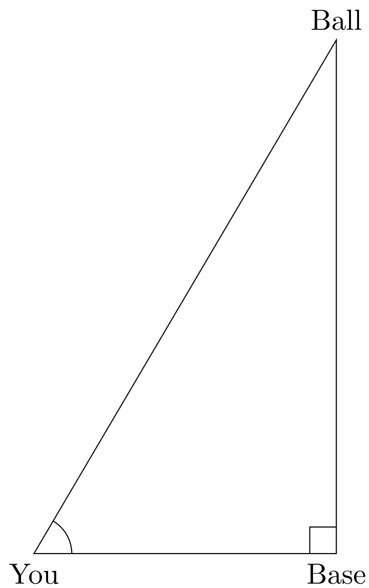
   A. f has local max at $x = 2$
   B. f has local min at $x = 2$
   C. Not enough info given to conclude whether f has any local extrema.
   D. f has an inflection point at $x = 2$.

7. (20 points) Differentiate the following functions, you do not need to simplify your answers. Use logarithmic differentiation if needed.

   (a) (8 points) $f(x) = \ln(\sin x) + \arctan(x^2)$

   (b) (12 points) $h(x) = (x^2 + 1)^{2x}$

8. (12 points) You are watching the Times Square Ball drop on New Year's Eve at a distance of 100m away from the base of the structure. The ball drops vertically at a constant rate, and as it does your angle of view changes as well. If the ball drops at a rate of 2 meters per second, then what is the rate of change of your angle of view when the angle is $\pi/4$ from the horizontal?



9. (12 points) Of all rectangles with a perimeter of 20 cm, which one has the maximum area?

   (a) Organize the given information with a picture and identify the variables.

   (b) What quantity (objective) needs to be optimized? What condition (constraint) needs to be satisfied?

   (c) Find a function for the quantity that is being optimized, and find its domain.

   (d) Use methods of calculus to find the absolute maximum or minimum value of the objective function on the domain.

   (e) The final answer is:

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