Math 180

Name (Print): Solutions

3/09/2016

Time Limit: 90 Minutes

NetID: ______________________

This exam contains 10 pages (including this cover page) and 10 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on the top of this page.

The following rules apply:

- You may not open this exam until you are instructed to do so.
- You are expected to abide by the University's rules concerning Academic Honesty.
- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

TA Name:______________________

Circle your instructor.

- Martina Bode
- Jenny Ross
- Michael Hull
- Evangelos Kobotis
- Bonnie Saunders

1. (10 points) Find an equation for the tangent line to the graph given by the equation

$$x^2 + \underbrace{2xy}_{\text{product}} - y^2 = 1$$

at the point $(1, 2)$.

implicit diff

$2x + 2y + 2xy' - 2yy' = 0$

@ $(1,2)$

$2 + 4 + 2y' - 4y' = 0$

$6 = 2y'$

$y' = 3$

$\Rightarrow$ slope $= 3$
point $(1,2)$

$y - 2 = 3(x - 1)$

$\boxed{y = 3x - 1}$

2. (12 points) Differentiate the following functions, you do not need to simplify your answers. Note that $y = \sin^{-1}(x)$ is the inverse function to $\sin(x)$, $-\pi/2 \leq x \leq \pi/2$.

(a) (6 points) $f(x) = \sec(\sqrt{x})$

$$f'(x) = \sec(\sqrt{x})\tan(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

(b) (6 points) $f(x) = \sin^{-1}(\ln x)$

$$f'(x) = \frac{1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x}$$

3. (10 points) Differentiate $f(x) = x^{\sqrt{x}}$.

use logarithmic differentiation

$$f(x) = x^{\sqrt{x}}$$

$$\ln f(x) = \sqrt{x} \cdot \ln x$$

$$\frac{1}{f(x)} \cdot f'(x) = \left(\frac{1}{2\sqrt{x}} \ln x + \sqrt{x} \cdot \frac{1}{x}\right)$$

$$f'(x) = \left(\frac{1}{2\sqrt{x}} \ln x + \frac{\sqrt{x}}{x}\right) \cdot x^{\sqrt{x}}$$

$$= \left(\frac{\ln x + 2}{2\sqrt{x}}\right) \cdot x^{\sqrt{x}}$$

4. (10 points) The height of a vertically moving object is given by the function:

$$h(t) = 2t^3 + 3t^2 - 36t + 100$$

Here, we measure time in seconds and height in feet.

(a) (4 points) Find a formula for the velocity $v(t)$, and the acceleration $a(t)$ of the moving object.

$v(t) = 6t^2 + 6t - 36$

$a(t) = 12t + 6$

(b) (5 points) If the object starts moving at $t = 0$, find at what point in time its velocity becomes 0.What is the acceleration of the object at that point in time; provide the correct units with your answer.

$v(t) = 0 \Rightarrow 6t^2 + 6t - 36 = 0$

$6(t+3)(t-2) = 0$

$\cancel{t = -3}$ or $\boxed{t = 2}$

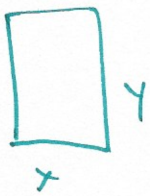
$a(2) = 12 \cdot 2 + 6 = 30 \text{ ft}/s^2$

(c) (1 point) What does the second derivative test tell you at that point in time that you found in part (b).

$\left.\begin{array}{l} h'(2) = 0 \\ h''(2) = 30 \end{array}\right\} \Rightarrow$ there is a local min @ $t=2$

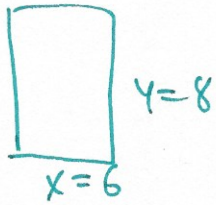
i.e. the height is at a min and the object starts to move back up at $t=2$.

5. (12 points) A rectangle in a video game changes dimensions as the game proceeds. When the height is 8 cm, and the width is 6 cm, the rate of change of its height is 3 cm/sec and the rate of change of its width is -2 cm/sec, find the rate of change of the rectangle's area at this moment.



Given: $\frac{dx}{dt} = -2 \quad \frac{dy}{dt} = +3$

Want: $\frac{dA}{dt}$ @



$A = xy$

$$\frac{dA}{dt} = x \cdot \frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{dA}{dt} = 6(+3) + 8(-2) = 2\ cm^2/sec$$

the area changes at $2cm^2/sec$

6. (12 points) Find the absolute maximum and minimum of $f(x) = \left(x^2 - 1\right)^3$ on $[0, 2]$. Show all your work.

$f'(x) = 3(x^2-1) \cdot 2x$

(i) $f' = 0 \Rightarrow x = 0$ $\underbrace{\cancel{x=-1}}_{\text{not in } [0,2]}$ and $x = +1$

(ii) $f' = \emptyset$

(iii) endpoints $x = 0$ & $x = 2$

Compare values of points of interest:

	$f(x)$
0	1
1	0 = absolute min value
2	27 = absolute max value

7. (8 points) Determine the interval(s) on which the function $f(x) = xe^{-x}$ is increasing and the interval(s) on which it is decreasing.

$f' = e^{-x} + (-1)xe^{-x}$
$= (1-x)e^{-x}$

sign of f': $+$ | $-$ at 1, $\nearrow$ 1 $\searrow$

$f' = 0 \Rightarrow x = 1$
$f' = \emptyset$ DNA

f increasing on $(-\infty, 1)$
f is decreasing on $(1, \infty)$

8. (8 points) Let $f(x) = \dfrac{x}{x^2+1}$, $f'(x) = \dfrac{1-x^2}{(x^2+1)^2}$, and $f''(x) = \dfrac{2x^3-6x}{(x^2+1)^3}$. $= \dfrac{2x(x^2-3)}{(x^2+1)^3}$

Determine the intervals of concavity and the inflection points of $f(x)$.

$f'' = 0$ $x = 0$
$x = \pm\sqrt{3}$

$f'' = \emptyset$ DNA

$2x$	$-$	$-$	$+$	$+$
x^2-3	$+$	$-$	$-$	$+$
x^2+1	$+$	$+$	$+$	$+$
f''	$-$	$+$	$-$	$+$
	$\frown$ $-\sqrt{3}$	$\smile$ 0	$\frown$ $\sqrt{3}$	$\smile$

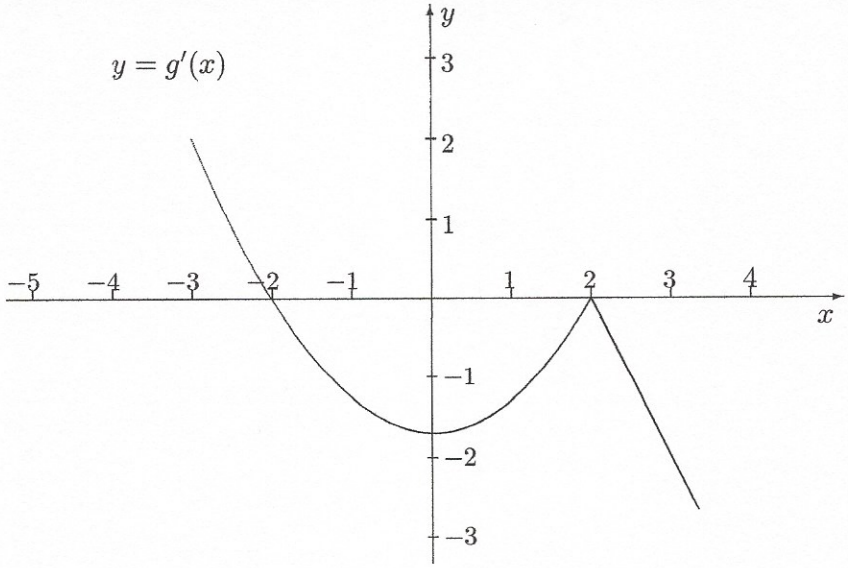
Concave down on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

concave up on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

points of inflections:

@ $x = -\sqrt{3}$
$x = 0$
$x = \sqrt{3}$

9. (8 points) The graph of the derivative function $y = g'(x)$ on the interval $[-3, 3]$ is shown below.



(a) (4 points) On what intervals is g increasing? decreasing? At what value(s) of x does g have a local maximum? local minimum?

g increasing when $g' > 0$: on $(-3,-2)$

g decreasing when $g' < 0$: on $(-2,2) \cup (2,3)$

@ $x=-2$ sign of g' $+ \mid -$; -2

local max

(b) (4 points) On what intervals is g concave upward? concave downward? Does g have any points of inflection?

g concave up when g' is increasing : on $(0,2)$

g concave down when g' is decreasing : on $(-3,0) \cup (2,3)$

sign of g'' $-$ $+$ $-$; -3 0 2 3

two points of inflection: @ $x=0$ & $x=2$

10. (10 points) Suppose that the function $f(x)$ has an unknown formula, but the following properties are known.

(a) Domain$= [0, 4]$.

(b) $f(0) = 0, f(1) = 1, f(2) = 2, f(3) = 3, f(4) = 2$.

(c) $f'(x) > 0$ for $x < 3$, and $f'(x) < 0$ for $x > 3$.

(d) $f''(x) > 0$ for $0 < x < 1$ and $x > 2$, and $f''(x) < 0$ for $1 < x < 2$.

Sketch a possible graph of $y = f(x)$.