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MATH 180 Exam 2
March 14, 2017

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: ______________________________

University Email: ______________________________

UIN: ______________________________

Circle your instructor's name: Shulman Steenbergen Thulin Zhang

TA's Name: ______________________________

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. (10 points) Find the following derivatives. DO NOT SIMPLIFY YOUR ANSWERS.

(a) $\frac{d}{dx}\left(\sin^{-1}(2x)\right)$ $= \frac{2}{\sqrt{1-(2x)^2}}$

(b) $\frac{d}{dx}\left(x^{\sqrt{x}}\right)$ [this is x to the $\sqrt{x}$ power]

$\ln y = \sqrt{x}\ \ln x$

$\frac{1}{y}\cdot y' = \frac{1}{2\sqrt{x}}\ln x + \frac{\sqrt{x}}{x}$

$y' = \left(\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}\right)\cdot x^{\sqrt{x}}$

2. (10 points) Find an equation of the line tangent to the graph of $y^2 + (y-x)^3 = 5$ at $(1, 2)$.

$2y\frac{dy}{dx} + 3(y-x)^2\left(\frac{dy}{dx} - 1\right) = 0$

@ (1,2) plug in x=1 & y=2

$4\frac{dy}{dx} + 3\underbrace{(2-1)^2}_{1}\left(\frac{dy}{dx} - 1\right) = 0$

$4\frac{dy}{dx} + 3\frac{dy}{dx} - 3 = 0 \Rightarrow 7\frac{dy}{dx} = +3 \Rightarrow \frac{dy}{dx} = \frac{3}{7}$

$y - 2 = \frac{+3}{7}(x-1)$

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3. (12 points) Consider the function $f(x) = 4x + \frac{1}{\sqrt{x}}$.

(a) State the domain of f.

Domain $= (0, \infty)$

(b) Find the absolute maximum and absolute minimum of f, or explain why one or both do not exist. Write your answers in the box below.

$f'(x) = 4 - \frac{1}{2}x^{-3/2}$

$= 4 - \frac{1}{2\sqrt{x^3}}$

sign of f' $\;($ 0 $-$ $\searrow$ $\frac{1}{4}$ $+$ $\nearrow$

plug in $x=1$ $f'(1)=3>0$

(plug in $x=\frac{1}{16}$ into $f'(\frac{1}{16}) = 4 - \frac{1}{2\sqrt{16^3}} < 0$)

$f'=0 \quad 4 = \frac{1}{2\sqrt{x^3}}$

$\sqrt{x^3} = \frac{1}{8}$

$\boxed{x = \frac{1}{4}}$

@ $x=\frac{1}{4}$ absolute min and

$f(\frac{1}{4}) = 4 \cdot \frac{1}{4} + \frac{1}{\sqrt{\frac{1}{4}}} = 1+2=3$

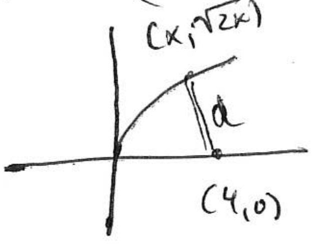
Since $x=0$ is not in the domain, there is no absolute max

Absolute Maximum: none Absolute Minimum: $(\frac{1}{4}, 3)$

or absolute min value $=3$

4. (10 points) Find the point (x, y) on the graph of $y = \sqrt{2x}$ nearest the point $(4, 0)$.

$f(x) = (\text{distance})^2 = (x-4)^2 + (\sqrt{2x})^2 = (x-4)^2 + 2x \quad D = (0, \infty)$



$f'(x) = 2(x-4) + 2$

$= 2x - 6$

$f'=0 \quad x=3$

$f' = \phi$ N/A

sign of f' $\;($ 0 $-$ $\searrow$ 3 $+$ $\nearrow$

absolute min @ $x=3$

& $y = \sqrt{2 \cdot 3} = \sqrt{6}$

note it is easier to find minimum $(\text{distance})^2$.

$\Rightarrow \boxed{(3, \sqrt{6})}$

5. (10 points) Oil spills from a ruptured tanker and spreads in a circle whose area increases at a constant rate of 8 square miles per hour.

(a) What is the size of the radius when the area is 4 square miles?

area $= \pi r^2 = 4 \Rightarrow r^2 = \frac{4}{\pi} \Rightarrow r = \frac{2}{\sqrt{\pi}}$

(b) How fast is the radius increasing when the area of the spill is 4 square miles? Make sure you include your units.

$\frac{dA}{dt} = 8 \qquad A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$8 = 2\pi \frac{2}{\sqrt{\pi}} \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{8}{2\pi} \cdot \frac{\sqrt{\pi}}{2} = \frac{2}{\sqrt{\pi}}$ miles per hour

6. (12 points) Consider the function $f(x) = \ln x$.

(a) Find the linear approximation of $f(x)$ at $x = 1$.

$f'(x) = \frac{1}{x} \Rightarrow f'(1) = 1$
$f(1) = \ln(1) = 0$

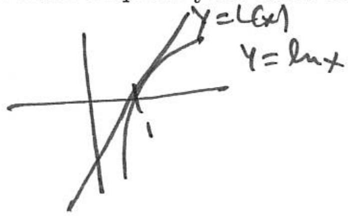
$L(x) = 0 + 1 \cdot (x-1)$

$L(x) = x - 1$

(b) Using your answer in part (a), estimate ln(2).

$\ln 2 \approx L(2) = 2 - 1 = 1$

(c) Determine whether the approximation in part (b) is an overestimate or underestimate to the actual value. Explain your answer.



OVER ESTIMATE

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7. (16 points) Consider the function $f(x) = x^4 - 4x^3$.

(a) In the spaces provided below write the intervals on which the function is increasing, decreasing, concave up, and concave down.

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

$f' = 0 \Rightarrow x = 0$ or $x = 3$

$f' = \emptyset$ N/A

sign of f': $-$ | $-$ | $+$ at 0, 3; ↘ 0 ↘ 3 ↗

decreasing on $(-\infty, 0) \cup (0, 3)$ or $(-\infty, 3)$

increasing on $(3, \infty)$

$f'' = 12x^2 - 24x$
$= 12x(x-2)$

sign of f'': $+$ | $-$ | $+$ at 0, 2

$f'' = 0$ $x = 0$ or $x = 2$

$f'' = \emptyset$ N/A

Concave up $(-\infty, 0) \cup (0, \infty)$

down $(0, 2)$

Increasing: $(3, \infty)$ Decreasing: $(-\infty, 3)$

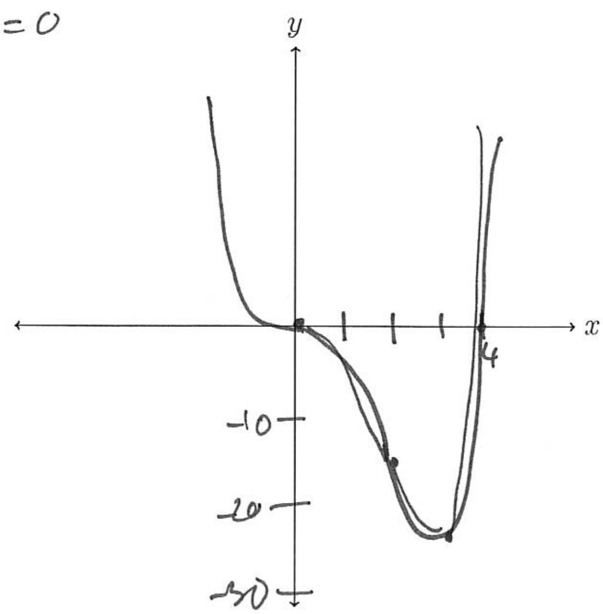
Concave Up: $(-\infty, 0) \cup (2, \infty)$ Concave Down: $(0, 2)$

(b) On the axes provided sketch a graph of f that reflects your answers in parts (a). Your graph should also label any x- or y-intercepts.

$f = 0$ $x^3(x-4) = 0$

$x = 0$ $x = 4$

x	f
0	0
2	$16 - 32 = -16$
3	$81 - 4 \cdot 27 = -27$



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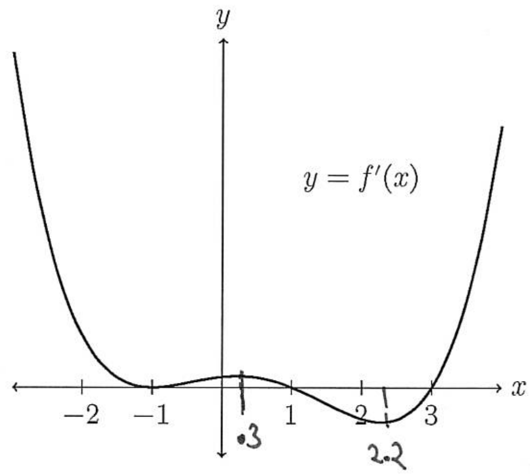
8. (10 points) Below is a graph of $f'(x)$, the DERIVATIVE of some function $f(x)$.



(a) State all of the critical points of f. For each critical point, classify it as a local minimum, local maximum, or neither.

$f'=0 \Rightarrow x=-1 \quad x=1 \quad x=3$

sign of f': $+$ | -1 | $+$ | 1 | $-$ | 3 | $+$

$\nearrow \quad -1 \quad \nearrow \quad 1 \quad \searrow \quad 3 \quad \nearrow$

@ $x=1$ local max

@ $x=3$ local min

@ $x=-1$ neither

(b) On which interval(s) is f concave down? Explain your answer.

f concave down when f' is decreasing

$(-\infty,-1) \cup (+0.3, 2.2)$

↑ ↑

approximations based on graph of f'

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9. (10 points) For each statement below, CLEARLY either circle "T" for TRUE or "F" for FALSE (if it is not clear which one you chose, it will be marked wrong). You do not need to justify your answer.

(a) T or F: If $f(x)$ is continuous on a closed interval $[a, b]$, then $f(x)$ must have an absolute maximum and an absolute minimum on $[a, b]$.

TRUE (extreme value theorem)

(b) T or F: If $f'(c) = 0$, then f has a minimum or maximum at $x = c$.

FALSE ($f(x) = x^3$ $f' = 3x^2$ $f'(0) = 0$)

(c) T or F: If $f(x) = x^3$, then f has an inflection point at $x = 0$.

TRUE

(d) T or F: If $f''(c) = 0$, then $f'(c) = 0$.

FALSE

(e) T or F: If $f'(c) = 0$ and $f''(c) > 0$, then f has a local maximum at $x = c$.

concave up

FALSE

local min

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THIS PAGE WAS LEFT BLANK INTENTIONALLY. YOU CAN USE IT FOR SCRATCH PAPER, BUT NOTHING ON THIS PAGE WILL BE GRADED.