MATH 180 Exam 2

March 19, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 10 pages (including this cover page) and 9 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

Name:_____

UIN:_____

UIC Email:_____

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Mercer (Tabes) Bridges
- Matthew Lee
- Jenny Ross

Write your Initials (First, Middle, Last)_____

1. (8 points) Find the equation of the tangent line to $x^2y^3 + x + y^2 = 1$ at $(1, -1)$

Write your Initials (First, Middle, Last)_____

2. (12 points) Differentiate the following functions, using logarithmic differentiation if needed. You do not need to simplify your answers

   (a) (6 points) $f(x) = \ln(x^3 - 6x^2 + 3)$

   (b) (6 points) $g(x) = \arcsin(2x^4 - 3)$

3. (8 points) Differentiate the following function, using logarithmic differentiation if needed. You do not need to simplify your answer.

$$h(x) = (x^3 - 9)^{10x}$$

Write your Initials (First, Middle, Last)_____

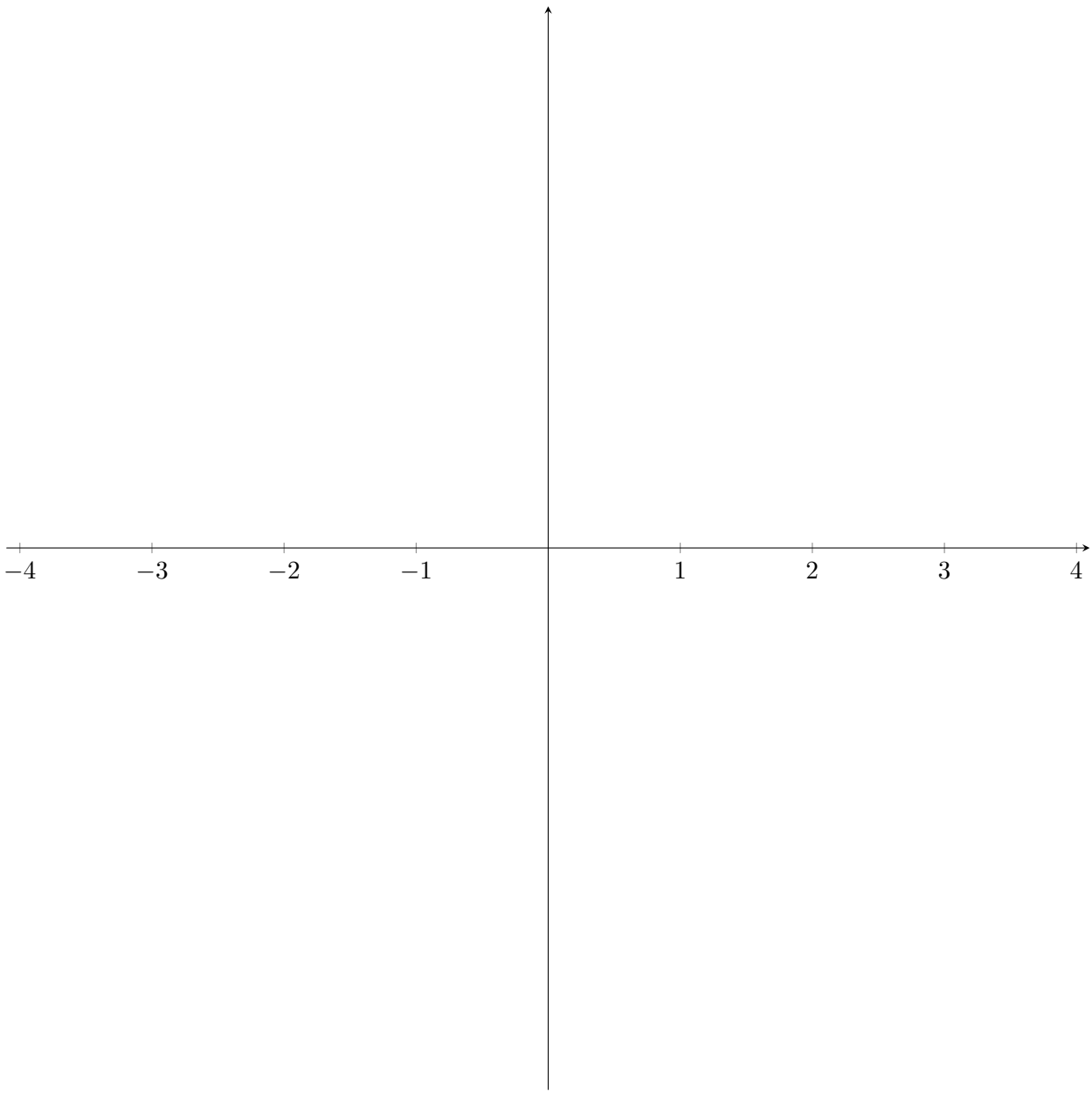
4. (29 points) Given $g(x) = \ln(4x^2 + 1)$ and its derivative, $g'(x) = \dfrac{8x}{4x^2 + 1}$, complete the following questions:

   (a) (8 points) On which intervals is $g(x)$ **increasing/decreasing**? Write, and label in **point form**, any **local maximums/minimums**. Justify with a **sign chart**.

   (b) (14 points) Find the intervals on which $g(x)$ is **concave up/down**. Verify with a **sign chart**. Write any **inflection point in point form**.

Problem 4 continued

(c) (7 points) Graph $g(x)$ and **label any maximum/minimum/inflection point with a large solid point**.



Write your Initials (First, Middle, Last)_____

5. (10 points) A police officer is informed that a speeding vehicle is 6 miles **West** of the main intersection of two highways. In an attempt to catch the speeder at this intersection, the police officer accelerates to try to arrive at the intersection, at the same time as the speeder.

   The officer is 8 miles **South** of the intersection, and they set their cruise control to 80 miles per hour, as they are informed the speeder is cruising at 70 miles per hour.

   How quickly is the distance between the police officer and speeder changing, when the police officer receives the call about the speeder?

   (a) (3 points) Draw a picture, illustrating their positions in relation to the intersection, when the officer receives the call, and label it.

   (b) (3 points) What is the relevant equation relating the labeled components in the picture?

   (c) (4 points) Finish the problem.

Write your Initials (First, Middle, Last)_____

6. (12 points) Find the absolute maximum and minimum values of the function $f(x) = x^3 - 3x^2 - 45x$ on the interval $[-5, 3]$. For which x-values do they occur?

7. (6 points) Find the x-values for which $g(x) = 3x^2 - 4x$ satisfies the Mean Value Theorem on $[-2, 6]$

Write your Initials (First, Middle, Last)_____

8. (8 points) Sketch the graph of a function satisfying the following conditions, on the interval $[-5, 4]$:

- $f'(x) > 0$ on $(-5, -2)$ and $(1, 4)$
- $f'(x) < 0$ on $(-2, 1)$
- $f(x)$ is continuous on $[-5, 4]$
- $f'(-2)$ is undefined
- $f''(x) > 0$ on $(-5, -2), (-2, 4)$
- $f'(1) = 0$, but $f(1)$ is not an absolute maximum, nor an absolute minimum

Write your Initials (First, Middle, Last)_____

9. (6 points) A farmer has 750 feet of fencing. He wants to build a fence enclosing a rectangular region and then divide that area in to 4 pens (in a row). What are the dimensions of the rectangular region with the largest (total) area?

This page can be used as scratch paper. It WILL NOT BE GRADED, so please SHOW YOUR WORK WITH YOUR PROBLEMS.