

ANSWERS

MATH 180 Exam 2

March 19, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. **YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR.** This exam contains 10 pages (including this cover page) and 9 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

Name: _____

UIN: _____

UIC Email: _____

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Mercer (Tabes) Bridges
- Matthew Lee
- Jenny Ross

Write your Initials (First, Middle, Last) _____

1. (8 points) Find the equation of the tangent line to $x^2y^3 + x + y^2 = 1$ at $(1, -1)$

use product rule

$$(2xy^3 + x^2 \cdot 3y^2 \frac{dy}{dx}) + 1 + 2y \frac{dy}{dx} = 0$$

@ $(1, -1) \rightarrow$ plug in $x=1$ & $y=-1$

$$2(1)(-1)^3 + (1)^2 \cdot 3(-1)^2 \frac{dy}{dx} + 1 + 2(-1) \frac{dy}{dx} = 0$$

$$-2 + 3 \frac{dy}{dx} + 1 - 2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 1$$

$$\left. \begin{array}{l} \text{slope} = 1 \\ \text{point } (1, -1) \end{array} \right\} \Rightarrow y + 1 = 1 \cdot (x - 1)$$

$$\boxed{y = x - 2}$$

Write your Initials (First, Middle, Last) _____

2. (12 points) Differentiate the following functions, using logarithmic differentiation if needed. You do not need to simplify your answers

(a) (6 points) $f(x) = \ln(x^3 - 6x^2 + 3)$

$$f'(x) = \frac{1}{x^3 - 6x^2 + 3} \cdot (3x^2 - 12x)$$

chain rule

(b) (6 points) $g(x) = \arcsin(2x^4 - 3)$

$$g'(x) = \frac{1}{\sqrt{1 - (2x^4 - 3)^2}} \cdot (8x^3)$$

3. (8 points) Differentiate the following function, using logarithmic differentiation if needed. You do not need to simplify your answer.

$$h(x) = (x^3 - 9)^{10x}$$

$$h'(x) = ? \text{ use logarithmic diff}$$

$$\ln h(x) = 10x \ln(x^3 - 9)$$

$$\frac{1}{h(x)} \cdot h'(x) = 10 \ln(x^3 - 9) + 10x \cdot \frac{3x^2}{x^3 - 9}$$

$$h'(x) = \left(10 \ln(x^3 - 9) + 10x \cdot \frac{3x^2}{x^3 - 9} \right) \cdot (x^3 - 9)^{10x}$$

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4. (29 points) Given $g(x) = \ln(4x^2 + 1)$ and its derivative, $g'(x) = \frac{8x}{4x^2 + 1}$, complete the following questions:

- (a) (8 points) On which intervals is $g(x)$ **increasing/decreasing**? Write, and label in **point form**, any **local maximums/minimums**. Justify with a **sign chart**.

critical points

$$g'(x) = \frac{8x}{4x^2 + 1} = 0 \Rightarrow x = 0$$

$$g'(x) \neq 0 \text{ N/A (since } 4x^2 + 1 \geq 1)$$

sign of g'

-	0	+
$\searrow$		$\nearrow$

g increasing on $(-\infty, 0)$,
decreasing on $(0, \infty)$

g has a local/global min @ $x=0$
no local max

- (b) (14 points) Find the intervals on which $g(x)$ is **concave up/down**. Verify with a **sign chart**. Write any **inflection point** in point form.

$$g''(x) = \frac{8(4x^2 + 1) - (8x)(8x)}{(4x^2 + 1)^2}$$

quotient rule $u = 8x$ $v = 4x^2 + 1$
 $u' = 8$ $v' = 8x$

$$= \frac{32x^2 + 8 - 64x^2}{(4x^2 + 1)^2}$$

$$g''(x) = \frac{8 - 32x^2}{(4x^2 + 1)^2}$$

sign of g''

-	+	-
$\searrow$	$\nearrow$	$\searrow$
$-\frac{1}{2}$	$\frac{1}{2}$	

$$g'' = 0 \quad 8 = 32x^2$$

$$\frac{1}{4} = x^2$$

$$x = \pm \frac{1}{2}$$

$$g'' \neq 0 \text{ N/A}$$

g concave up on $(-\frac{1}{2}, \frac{1}{2})$,
concave down on $(-\infty, -\frac{1}{2})$ and $(\frac{1}{2}, \infty)$

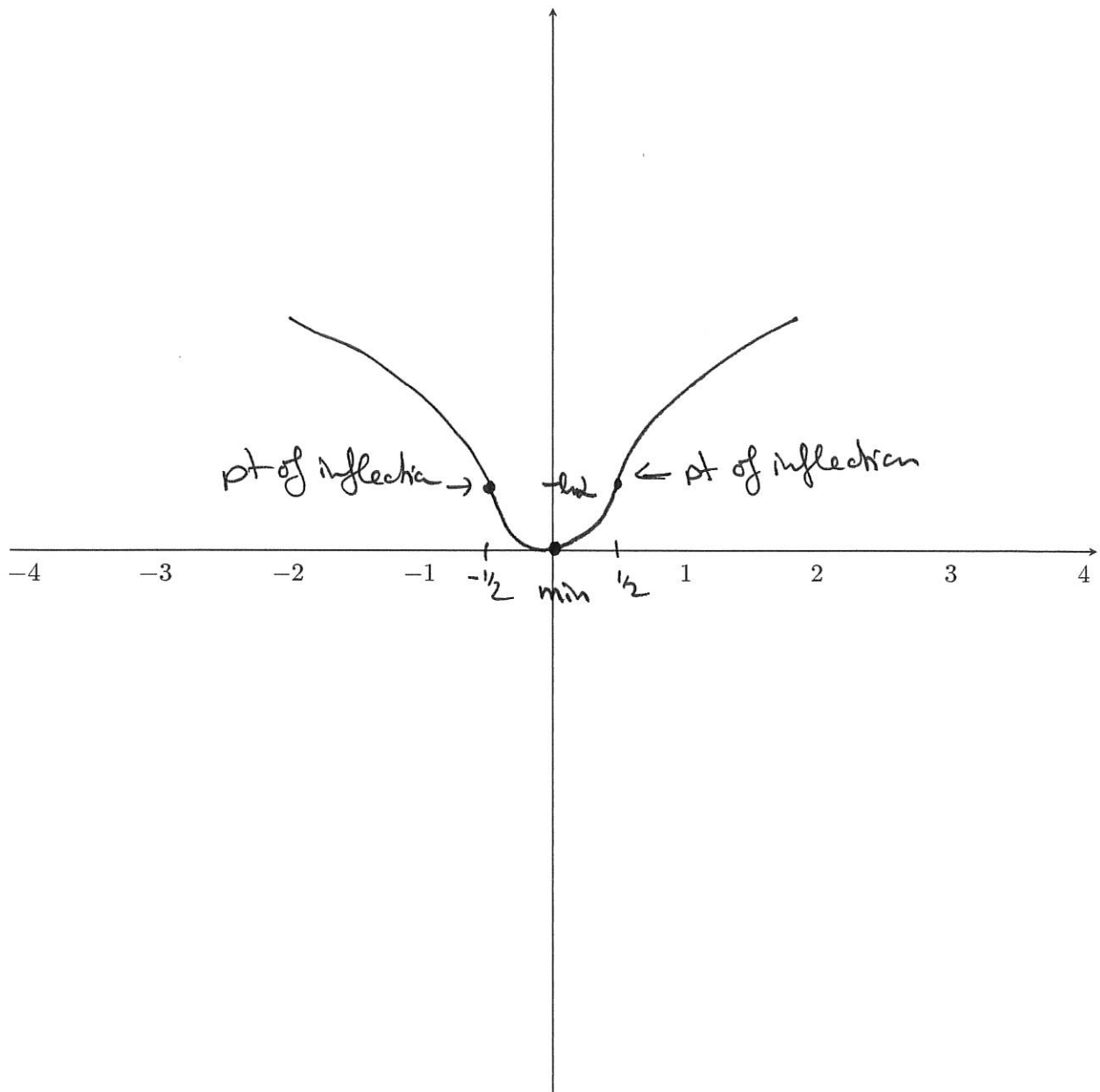
g has two points of inflection

@ $x = -\frac{1}{2}$ $y = \ln(4(\frac{1}{4}) + 1) = \ln 2$ $(-\frac{1}{2}, \ln 2)$

@ $x = \frac{1}{2}$ $y = \ln 2$ $(\frac{1}{2}, \ln 2)$

Problem 4 continued

- (c) (7 points) Graph $g(x)$ and label any maximum/minimum/inflection point with a large solid point.



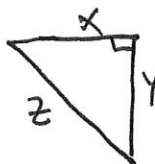
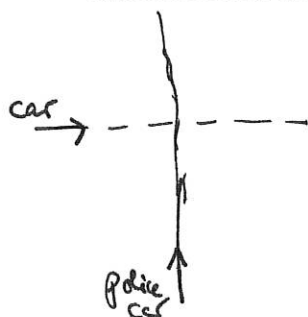
Write your Initials (First, Middle, Last) _____

5. (10 points) A police officer is informed that a speeding vehicle is 6 miles **West** of the main intersection of two highways. In an attempt to catch the speeder at this intersection, the police officer accelerates to try to arrive at the intersection, at the same time as the speeder.

The officer is 8 miles **South** of the intersection, and they set their cruise control to 80 miles per hour, as they are informed the speeder is cruising at 70 miles per hour.

How quickly is the distance between the police officer and speeder changing, when the police officer receives the call about the speeder?

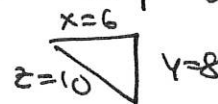
- (a) (3 points) Draw a picture, illustrating their positions in relation to the intersection, when the officer receives the call, and label it.



Given: $\frac{dx}{dt} = -70$

~~Let~~ $\frac{dy}{dt} = -80$

want $\frac{dz}{dt}$ @ $x=6$ & $y=8$



- (b) (3 points) What is the relevant equation relating the labeled components in the picture?

$$x^2 + y^2 = z^2$$

- (c) (4 points) Finish the problem.

$$x \frac{dx}{dt} + y \frac{dy}{dt} = z \frac{dz}{dt}$$

$$(6)(-70) + (8)(-80) = 10 \cdot \frac{dz}{dt}$$

$$-1060 = 10 \frac{dz}{dt}$$

$$-106 \text{ mph} = \frac{dz}{dt}$$

The distance is decreasing at a rate of 106 mph.

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6. (12 points) Find the absolute maximum and minimum values of the function $f(x) = x^3 - 3x^2 - 45x$ on the interval $[-5, 3]$. For which x -values do they occur?

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 45 \\ &= 3(x^2 - 2x - 15) \\ &= 3(x-5)(x+3) \end{aligned}$$

critical points

$$f' = 0 \quad \cancel{x=5} \text{ not in the interval}$$

$$x = -3 \quad \checkmark$$

$$f' = 0 \quad \text{N/A}$$

Compare values:

x	$f(x)$
-5	25
-3	81 = \text{max value @ } x = -3
3	-135 = \text{min value @ } x = 3

7. (6 points) Find the x -values for which $g(x) = 3x^2 - 4x$ satisfies the Mean Value Theorem on $[-2, 6]$

g continuous & differentiable on $[-2, 6]$ ✓

by MVT there is a c with

$$g'(c) = \frac{g(6) - g(-2)}{6 - (-2)} = \frac{84 - 20}{8} = 8$$

$$g'(c) = 6c - 4 = 8$$

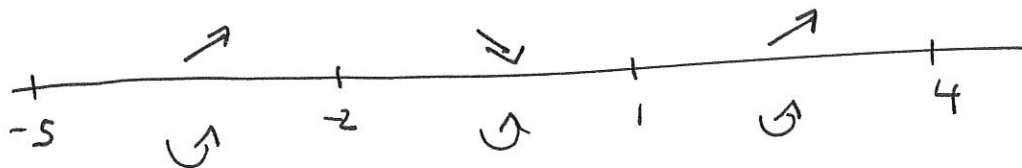
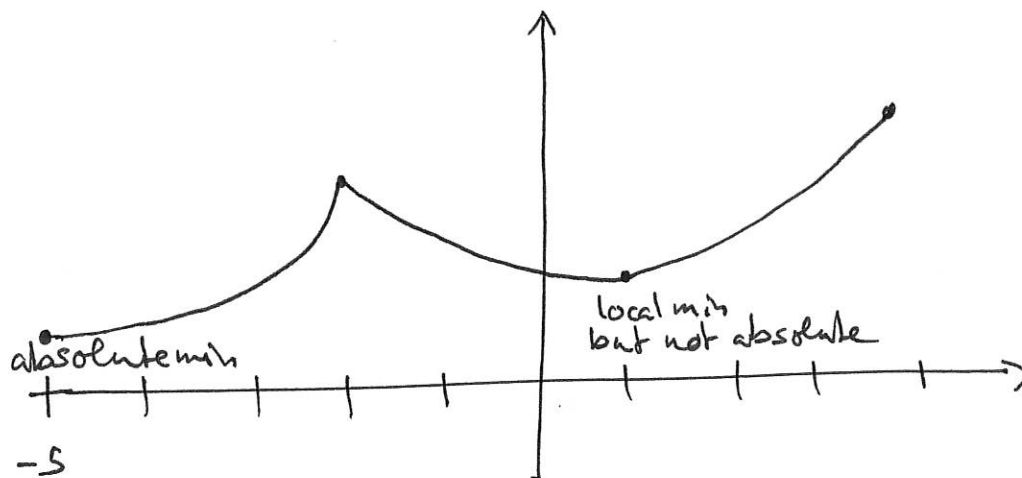
$$6c = 12$$

$$\boxed{c = 2}$$

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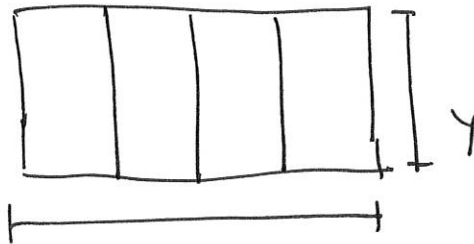
8. (8 points) Sketch the graph of a function satisfying the following conditions, on the interval $[-5, 4]$:

- $f'(x) > 0$ on $(-5, -2)$ and $(1, 4)$ *inc*
- $f'(x) < 0$ on $(-2, 1)$ *dec*
- $f(x)$ is continuous on $[-5, 4]$ *no break*
- $f'(-2)$ is undefined *not diff (corner)*
- $f''(x) > 0$ on $(-5, -2), (-2, 4)$ *concave up*
- $f'(1) = 0$, but $f(1)$ is not an absolute maximum, nor an absolute minimum
*flattens at $x=1$
but has to be a local min!*



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9. (6 points) A farmer has 750 feet of fencing. He wants to build a fence enclosing a rectangular region and then divide that area into 4 pens (in a row). What are the dimensions of the rectangular region with the largest (total) area?



Find largest area $A = x \cdot y$ subject to $2x + 5y = 750$

$$y = \frac{750 - 2x}{5}$$

$$A = x \cdot \frac{750 - 2x}{5}$$

$$f(x) = x \cdot \frac{750 - 2x}{5} = 150x - \frac{2}{5}x^2 \quad \text{Domain} = [0, 375]$$

(largest x-value when $y = 0$)

$$f'(x) = 150 - \frac{4}{5}x = 0$$

$$\begin{aligned} 2x &= 750 \\ x &= 375 \end{aligned}$$

$$\frac{4}{5}x = 150$$

$$x = 150 \cdot \frac{5}{4} = \frac{375}{2}$$

Sign of f'

0	+	-	375
	$\frac{375}{2}$		

absolute max

$$@ x = \frac{375}{2}$$

dimensions are $x = \frac{375}{2} \text{ ft}$, $y = \frac{750 - 375}{5} = \frac{375}{5} \text{ ft}$

This page can be used as scratch paper. It WILL NOT BE GRADED, so please SHOW YOUR WORK WITH YOUR PROBLEMS.