DO NOT WRITE ABOVE THIS LINE!!

MATH 181 Final Exam
December 8, 2016

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: ______________________________

University Email: ______________________________

UIN: ______________________________

Circle your instructor's name: Bode Lesieutre

TA's Name: ______________________________

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

DO NOT WRITE ABOVE THIS LINE!!

Question 1. *For each of the following questions, circle the correct answer. No explanation is necessary. No partial credit will be given.*

1. *(6 points) Find the derivative of the function:* $g(x) = \int_1^{\tan x} e^{t^2} dt$

   (a) $g'(x) = 2xe^{\tan^2(x)}$

   (b) $g'(x) = \sec^2 x e^{\tan^2 x}$

   (c) $g'(x) = \sec^2(x)e^{\tan^2 x} \sec x \tan x$

   (d) $g'(x) = \sec^2(x)e^{\tan^2 x} - \sec^2(1)e^{\tan^2(1)}$

2. *(6 points) Some values for a function* $y = f(t)$ *are given in the table below. Use Simpson's Rule to estimate the definite integral* $\int_0^{24} f(t)dt$.

t	0	6	12	18	24
$f(t)$	1/2	3/4	1/2	3/8	1/2

   (a) $\frac{1}{2} \cdot \left(\frac{1}{2} + 2 \cdot \frac{3}{4} + 2 \cdot \frac{1}{2} + 2 \cdot \frac{3}{8} + \frac{1}{2}\right)$

   (b) $\frac{6}{3} \cdot \left(\frac{1}{2} + 4 \cdot \frac{3}{4} + 2 \cdot \frac{1}{2} + 4 \cdot \frac{3}{8} + \frac{1}{2}\right)$

   (c) $\frac{1}{3} \cdot \left(\frac{1}{2} + 4 \cdot \frac{3}{4} + 2 \cdot \frac{1}{2} + 4 \cdot \frac{3}{8} + \frac{1}{2}\right)$

   (d) $\frac{6}{2} \cdot \left(\frac{1}{2} + 4 \cdot \frac{3}{4} + 2 \cdot \frac{1}{2} + 4 \cdot \frac{3}{8} + \frac{1}{2}\right)$

DO NOT WRITE ABOVE THIS LINE!!

Question 2. *(6 points) Given the matrix* $A = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$ *and the vectors:*

$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ *and* $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

One of these vectors is an eigenvector, which vector and what is its eigenvalue?

Question 3. *(12 points) Use partial fractions to evaluate the integral:*

$$\int \frac{(5x+1)\ dx}{x^2+x-2}$$

DO NOT WRITE ABOVE THIS LINE!!

Question 4. *(20 points) Evaluate the following integrals.*

(a) (10 points) $\displaystyle\int_0^{\sqrt{\pi}} 6\theta\sin(\theta^2+\pi)d\theta$

(b) (10 points) $\displaystyle\int (2x+1)\ln x \; dx$

DO NOT WRITE ABOVE THIS LINE!!

Question 5. *(12 points) Rewrite the following improper integral as limit and check whether or not the integral converges or diverges. If it converges, evaluate the integral.*

$$\int_0^4 \frac{4}{x-4}\,dx$$

DO NOT WRITE ABOVE THIS LINE!!

Question 6. *(16 points) Let R be the region enclosed by the parabolas $y = x^2$ and $y = 8 - x^2$.*

(a) (8 points) Sketch the region and set up the integral for the area of this region.

(b) (8 points) Set up the integral for the volume of the solid obtained by rotating the region R about the x-axis.

DO NOT WRITE ABOVE THIS LINE!!

Question 7. *(10 points) A cylindrical tank measures* 30 *ft high and* 20 *ft in diameter. It is one third filled with kerosene weighing approximately 50 lb/ft* ^{3}*. Set up the integral for the work done to pump all the kerosene out over the top of the tank.*

Question 8. *(10 points) Find the Taylor polynomial of degree 3 centered about* $a = \pi$ *for* $f(x) = \sin x$*.*

DO NOT WRITE ABOVE THIS LINE!!

Question 9. *(20 points) Test the following series for convergence or divergence. Justify your answer!*

(a) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+2}$

(b) (8 points) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{3^{n-1}}$

(c) (4 points) $\displaystyle\sum_{n=2}^{\infty} \frac{n+1}{n-1}$

DO NOT WRITE ABOVE THIS LINE!!

Question 10. *(10 points) Does the following series convergence absolutely, conditionally, or does it diverge? Justify your answer!*

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

DO NOT WRITE ABOVE THIS LINE!!

Question 11. *(16 points) Find the radius and the interval of convergence for the following power series. Justify your answer!*

$$\sum_{k=1}^{\infty} \frac{(x-3)^k}{k \cdot 6^k}$$

DO NOT WRITE ABOVE THIS LINE!!

Question 12. *(20 points)*
(a) (10 points) Find a power series representation and the interval of convergence for the function:

$$f(x) = \frac{1}{1 - x^4}$$

(You do not need to find a summation formula, just write out the addition or subtraction of the first 5 non-zero terms.)

(b) (6 points) Use the methods or results from part (a) to find a power series representation for the function:

$$g(x) = \frac{x}{1 - x^4}$$

(c) (4 points) Use the first three non-zero terms of the series you found in part (b) to approximate the indefinite integral:

$$\int g(x)\ dx$$

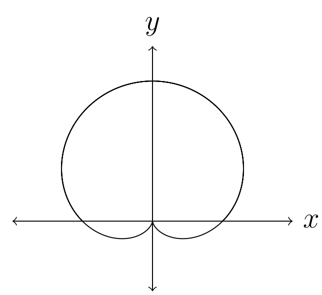
DO NOT WRITE ABOVE THIS LINE!!

Question 13. *(10 points) For the parametric curve given by* $x = 2t$ *and* $y = 4t^2 + 1$

(a) (5 points) Determine dy/dx *in terms of* t *and find an equation of the tangent line at* $t = 1$.

(b) (5 points) Eliminate the parameter t *and make a sketch of the curve showing the tangent line at the point corresponding to* $t = 1$.

Question 14. *(10 points) Consider the polar curve* $r = 1 + \sin\theta$ *graphed below.*



(a) (5 points) Find the rectangular coordinates x *and* y *for the point on the graph with* $\theta = \pi/4$.

(b) (5 points) Set up the integral for the area bounded by this curve in quadrant 1.

DO NOT WRITE ABOVE THIS LINE!!

Question 15. *(16 points) Consider the linear system:*

$$\begin{aligned} x + y &= 4 \\ x - y &= 2 \end{aligned}$$

(a) (4 points) Write down the augmented matrix for the linear system.

(b) (3 points) Translate this system to the format: $A \cdot \mathbf{x} = \mathbf{b}$.

(c) (4 points) Find the inverse matrix A^{-1} *to* A.

(d) (4 points) Use the inverse matrix from part (b) to solve for $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$.

(e) (1 point) Double check your answer by plugging in x *and* y *to the linear system above.*