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MATH 181 Final Exam
May 4, 2017

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: ______________________________

University Email: ______________________________

UIN: ______________________________

Circle your instructor's name: Boester Cappetta Steenbergen

TA's Name: ______________________________

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

SECTION 1: MULTIPLE CHOICE
For each multiple choice question below (Problems 1-6), show all necessary work in addition to circling the correct response. Each question will be graded in terms of both your circled choice and the work provided. In order to earn full credit, you must have a clearly circled answer and written work to support that answer.

1. (10 points) Evaluate the following integral: $\int_0^{\frac{\pi}{2}} x \sin x \, dx$
   (a) $\frac{1}{3}$
   (b) $\frac{1}{2}$
   (c) $\frac{2}{3}$
   (d) $\frac{3}{4}$
   (e) 1

2. (9 points) The horizontal base of a cylindrical tank is a circular disk of radius 1 m (meter) and its height is 4 m. It is half filled with a liquid that weighs $3 \text{ N/m}^3 = (\text{density } 0.3 \text{ kg/m}^3)(\text{gravity } 10 \text{ m/s}^2)$. How much work, in Nm, is needed to pump out that liquid over the top rim?
   (a) 6π
   (b) 8π
   (c) 12π
   (d) 15π
   (e) 18π

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3. (10 points) Evaluate the following integral: $\int_0^{\frac{\pi}{2}} \sin^3 x \; dx$

   (a) 0

   (b) $\dfrac{1}{3}$

   (c) $\dfrac{2}{3}$

   (d) 1

   (e) $\dfrac{4}{3}$

4. (6 points) The region bounded by the graph $y = 1 + x^2$ and the line $y = 2$ is rotated about the x-axis to form a solid. (For supporting work to earn full credit, sketch a graph of the enclosed region and label intersection points.) The volume of the solid is given by:

   (a) $\displaystyle\int_1^2 2\pi \left[2^2 - \left(1 + x^2\right)^2\right] dx$

   (b) $\displaystyle\int_1^2 \pi \left[2^2 - \left(1 + x^2\right)^2\right] dx$

   (c) $\displaystyle\int_{-1}^1 2\pi \left[2^2 - \left(1 + x^2\right)^2\right] dx$

   (d) $\displaystyle\int_{-1}^1 \pi \left[2^2 - \left(1 + x^2\right)^2\right] dx$

   (e) $\displaystyle\int_1^2 \pi \left(1 - x^2\right)^2 dx$

5. (10 points) Given $f(x) = \dfrac{1}{1-x^2}$, use Taylor/Maclaurin series to compute $f^{(4)}(0)$, the fourth derivative of $f(x)$ evaluated at $x = 0$.

   (a) 0

   (b) 4

   (c) 8

   (d) 12

   (e) 24

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6. (12 points) Find the interval of convergence for the following series: $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{k+1} x^{2k}$

(a) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(e) $\left(-\frac{1}{2}, \frac{1}{2}\right]$

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SECTION 2: FREE RESPONSE
For each free response question below (Problems 7-15), show all necessary work to earn full credit.

7. (16 points) Evaluate the following integrals:

   (a) $\displaystyle\int \frac{x+3}{x^2+6x+1}\,dx$

   (b) $\displaystyle\int \frac{x+8}{(x+1)\,(2x-5)}\,dx$

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8. (12 points) Evaluate the following integral: $\displaystyle\int_1^\infty \frac{1}{x^2}\, dx$. Show all work.

9. (10 points) Construct a Taylor polynomial of order 3 centered at $x = 1$ for $f(x) = \sqrt{x}$.
DO NOT SIMPLIFY YOUR ANSWER.

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10. (16 points) Determine whether each of the following series converge or diverge. Justify your answer and name the test(s) used.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{2^{2k}}{3^{k+1}}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{4k^2 + 7k - 1}{11k^2 + 17}$

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11. (20 points)

(a) Does $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2k+5}$ converge? Justify your answer and name the test(s) used.

(b) Does $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{2k+5}$ converge? Justify your answer and name the test(s) used.

(c) Does $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{2k+5}$ converge absolutely or conditionally? Briefly explain why.

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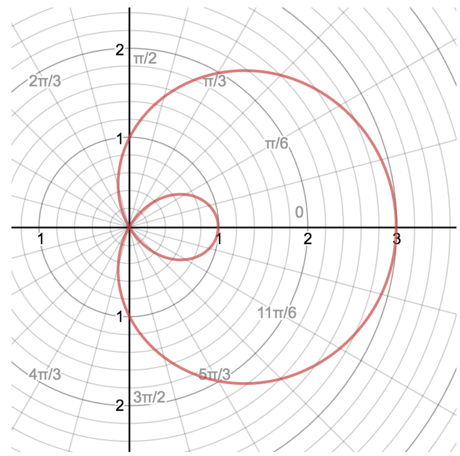
12. (16 points) Given the parametric equation $x = 5\cos t$ and $y = 2\sin t$ for $0 \leq t \leq 2\pi$:

(a) Sketch a graph of the curve.

(b) Find the slope of the tangent line at $t = \dfrac{\pi}{6}$.

13. (22 points) On the right is a graph of $r = 1 + 2\cos\theta$:

(a) Translate the point on the curve at $\theta = \dfrac{\pi}{3}$ into Cartesian coordinates.



(b) Find the slope of the tangent line at $\theta = \dfrac{\pi}{2}$.

(c) Set up but DO NOT SOLVE the integral to find the area of the inner loop.

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14. (18 points) For the system of equations

$$2x + 3y = 4$$
$$4x + y = -2$$

(a) Solve the system of equations by setting up an augmented matrix and reducing to upper triangular form.

(b) Set up the system of equations as $A\mathbf{x} = \mathbf{b}$. Find the inverse matrix A^{-1}. Then use the inverse matrix to solve the system of equations.

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15. (13 points) Given the matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$:

(a) Determine which of the following are eigenvectors of A.

i. $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

ii. $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

iii. $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

iv. $\mathbf{v}_4 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

v. $\mathbf{v}_5 = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$

(b) For each vector in part (a) that is an eigenvector, specify the appropriate eigenvalue.