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MATH 181 Final Exam
May 10, 2018

Directions. Fill in each of the boxes below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

- Circle your instructor: Cappetta Diep Shulman
- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- You must show all of your work and justify all answers.
- A solution for one problem may not go on another page.
- If you are asked to calculate an improper integral, make sure you justify your answer if it converges or diverges.

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1. (points) Calculate $\displaystyle\int_0^{\pi/4} \sin(2x)\ dx$.

2. (points) Calculate $\displaystyle\int (z+1)\sqrt{z+7}\ dz$.

3. (points) Calculate $\displaystyle\int_0^{\pi/4} x\sin(2x)\ dx$.

4. (points) Find the interval convergence of $\displaystyle\sum_{k=0}^{\infty} \frac{x^{3k}}{27^k}$. Remember to test the endpoints.

5. (points) Calculate $\displaystyle\int x^2 \ln(x)\ dx$.

6. (points) Calculate $\displaystyle\int \frac{13x+4}{(x+3)(2x-1)}\ dx$.

7. (points) The hyperbolic cosine function, $\cosh x$, is defined by $\cosh x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$. Find the first five nonzero terms of the Maclaurin series for $f(x) = \cosh x$.

8. (points) Let $g(x) = \dfrac{12}{4+x}$.

   (a) Using the sigma symbol below, express $g(x)$ as a series entered at 0.

   $$\sum_{k=0}^{\infty}$$

   (b) Find the radius of convergence of the series.

9. (points) Consider the system of equations

   $$\begin{cases} 3x - 2y = 5 \\ 4x + 7y = 1 \end{cases}$$

   (a) Rewrite the system as a matrix equation $A\mathbf{x} = \mathbf{b}$.

   (b) Find A^{-1}.

   (c) Use A^{-1} to solve the system. Show your work.

10. (points) Let $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$.

    (a) Find the eigenvalues of A.

    (b) Find an eigenvector for each of the eigenvalues you found in (a).

11. (points) Find the area bounded by the curves $y = 1 + x^2$ and $y = 3 + x$.

12. (points) Consider the line segment from the point $(0, 2)$ to the point $(3, 8)$. Fill in the parametrization below, making sure to also fill in the interval for t.

$$x(t) = \underline{\hspace{3cm}},\quad y(t) = \underline{\hspace{3cm}} \qquad\qquad \leq t \leq$$

13. (points) (a) Convert the two rectangular (Cartesian) points to polar coordinates with $r > 0$ and $0 \leq \theta < 2\pi$.

$(-3, 3)$ and $(-\sqrt{3}, 1)$

(b) Convert the two polar points to rectangular (Cartesian) coordinates.

$\left(5, \dfrac{\pi}{4}\right)$ and $\left(-3, -\dfrac{7\pi}{4}\right)$

(c) Sketch the set of points $\left\{(r, \theta) : 2 \leq r \leq 5,\ \dfrac{\pi}{2} \leq \theta \leq \pi\right\}$.

14. (points) Determine if the following series converge or diverge.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k 5^k}{2^{3k}}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{k - k^3}{3k^4 + k^2 + 1}$

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