MATH 181 Final Exam

May 9, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 12 pages (including this cover page) and 14 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

TA Name:_____

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Martina Bode
- Robert Cappetta
- Vi Diep
- John Steenbergen

DO NOT WRITE ABOVE THIS LINE

1. (24 points) Evaluate the following integrals. Show all your work!

   (a) (8 points) $\int_0^1 \left(x^2 e^{x^3+1}\right) dx$

   (b) (8 points) $\int \left(3x \sin(4x)\right) dx$

   (c) (8 points) $\int \left(2\cos^3(x)\right) dx$

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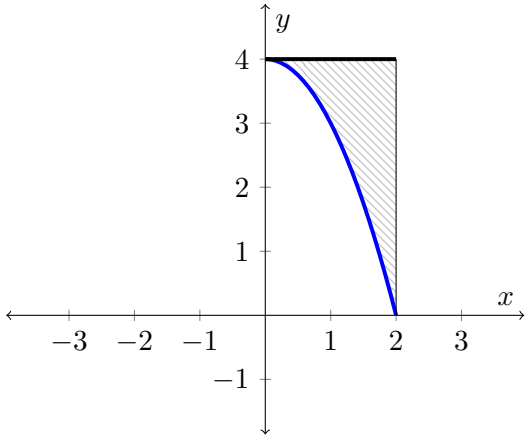
2. (10 points) Use partial fractions to evaluate the following integral. Show all your work!

$$\int \frac{6x-6}{x^2-9}\,dx$$

3. (12 points) Determine whether the following improper integral diverges or converges. If it converges, find its value. Show all your work!

$$\int_0^\infty e^{-5x}\,dx$$

4. (12 points) The region R below is the region enclosed by the parabola $y = 4 - x^2$, $x = 2$, and $y = 4$.



(a) (6 points) Find the area of the region R.

(b) (6 points) Set up the integral for the volume of the solid obtained by rotating the region R about the line $y = 4$. **Do not evaluate the integral!**

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5. (12 points) A fish tank has a rectangular base 3 meter long and 2 meter wide. The sides are 1 meter high, and the tank is full of water. Set up the integral for the work it takes to empty the tank by pumping the water out over the top of the tank. The density of water is 1000 kg/m ^{3}, and the acceleration due to gravity is $g = 9.8\text{m/s}^2$. **Do not evaluate the integral!**

6. (10 points) Find the fourth order Taylor Polynomial for

$$f(x) = 1 + x^3 + \frac{1}{x}$$

centered at $a = 1$. Do not simplify!

7. (18 points) Find the interval of convergence of the following power series.

$$\sum_{k=1}^{\infty} \frac{(x+2)^k}{k \cdot 3^k}$$

Show all your work, and circle the correct answer below. Name all tests that you are using.

Circle the correct interval:

(a) $[-1, 5]$ (b) $[-1, 5)$ (c) $[-5, 1]$ (d) $(-5, 1)$ (e) $[-5, 1)$

DO NOT WRITE ABOVE THIS LINE

8. (24 points) Determine whether the following series converge or diverge. Justify your answers and state which tests you are using.

(a) (8 points) $\displaystyle\sum_{k=2}^{\infty} \frac{k}{k^2-1}$

(b) (8 points) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k}+1}{k^2}$

(c) (8 points) $\displaystyle\sum_{k=1}^{\infty} \frac{7k+1}{2k+5}$

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9. (16 points) (a) (5 points) Does $\sum_{k=4}^{\infty} \frac{1}{k^3}$ converge? Name the test(s) you are using, and show all your work.

(b) (6 points) Does $\sum_{k=4}^{\infty} \frac{(-1)^k}{k^3}$ converge? Name the test(s) you are using, and show all your work.

(c) (5 points) Does $\sum_{k=4}^{\infty} \frac{(-1)^k}{k^3}$ converge absolutely, converge conditionally, or diverge?

10. (10 points) Consider the curve $x = 2\sin t$, $y = 3 + \cos t$ for $0 \leq t \leq 2\pi$. Find the equation of the tangent line when $t = \frac{\pi}{4}$.

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11. (18 points) Given the following power series representation

$$\arctan(x) = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k+1}}{2k+1}$$

answer the following questions.

(a) (6 points) List at least the first four non-zero terms of this series.

(b) (6 points) List at least the first three non-zero terms of the power series representing the function:

$$\frac{\arctan(x) - x}{x^3}$$

(c) (6 points) Use your results in part (b) (and not L'Hôpital's Rule) to evaluate the limit:

$$\lim_{x \to 0} \frac{\arctan(x) - x}{x^3}$$

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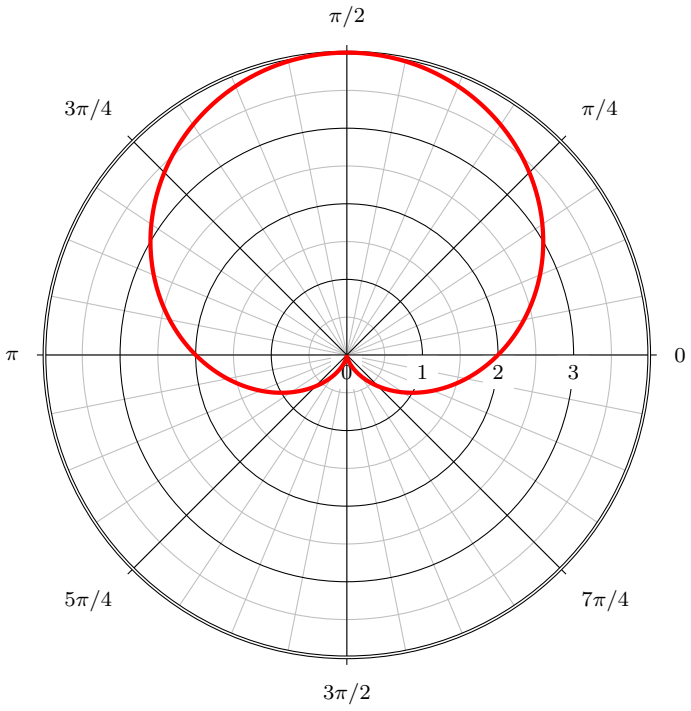
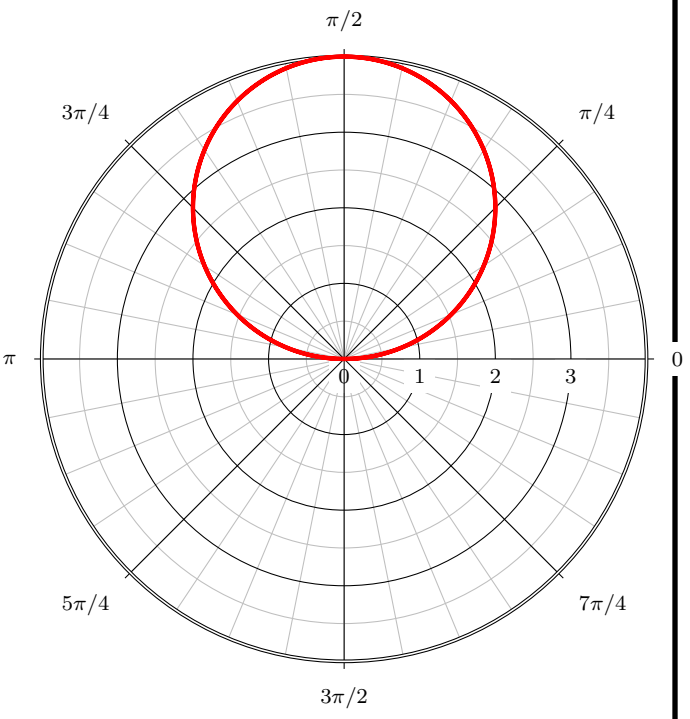
12. (10 points) Consider the polar curve $r = 4\sin(\theta)$ for $0 \leq \theta \leq 2\pi$.

(a) (6 points) Fill in the table below.

θ	r
0	
$\pi/4$	
$\pi/2$	
π	
$3\pi/2$	
2π	

(b) (4 points) Circle the polar graph below that represents the polar curve $r = 4\sin(\theta)$ for $0 \leq \theta \leq 2\pi$.





13. (12 points) Consider the matrix $A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$. For each vector below, determine if it is an eigenvector for A or not (make sure to justify your work). If it is an eigenvector, find its associated eigenvalue.

(a) (4 points) $\vec{v_1} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(b) (4 points) $\vec{v_2} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

(c) (4 points) $\vec{v_3} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$

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14. (12 points) Consider the following linear system

$$\begin{cases} x + 3y = 7 \\ 2x + 5y = 13 \end{cases}$$

(a) (3 points) Write the system above as an augmented matrix.

(b) (3 points) Perform elementary row operations to solve the system. Make sure you write down each of your elementary operations.

(c) (3 points) Write the 2×2 coefficient matrix for this system and call it A. Find $\det(A)$ and A^{-1}.

(d) (3 points) Express the system as a matrix equation $A\mathbf{v} = \mathbf{b}$. Solve this system using matrix inverses. Note that the answer should match your answer in part (b).