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MATH 181 Exam 1
September 28, 2016

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: SOLUTIONS

University Email: ________________

UIN: ________________

Circle your instructor's name: Bode Lesieutre

TA's Name: ________________

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. (10 points) A car was accelerating from $t = 0$ to $t = 8$ *sec*. Here is the speedometer data in m/sec.

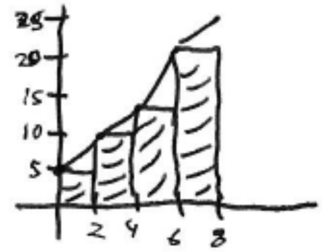
t sec	0	2	4	6	8
v m/sec	5	10	14	21	25

Approximate the car's displacement using:

(a) (6 points) A left endpoint approximation with $n = 4$. You do not need to simplify your answer! Is this an overestimate or underestimate? Justify your answer.

$$s(8) = \int_0^8 v(t)\,dt.$$

$$LHS = (5+10+14+21)\cdot 2 = \boxed{100}$$



(b) (4 points) Simpson's rule with $n = 2$. You do not need to simplify your answer!

$n=2$ means we only use $\overset{x_0}{0}, \overset{x_1}{4}, \overset{x_2}{8}$. The rule gives

$$v(8) \approx (1\cdot 5 + 4\cdot 14 + 1\cdot 25)\frac{4}{3}^{\leftarrow \Delta x} = \boxed{\frac{344}{3}.}$$

2. (10 points) Find the length of the curve $y = \frac{2}{3}x^{3/2}$ for x between $x = 0$ and $x = 1$.

Arc length is $\int_a^b \sqrt{1+f'(x)^2}\,dx.$

$f(x) = \frac{2}{3}x^{3/2}$

$f'(x) = x^{1/2}$

$1 + f'(x)^2 = 1 + x$

$$L = \int_0^1 \sqrt{x+1}\,dx = \frac{2}{3}(x+1)^{3/2}\Big|_0^1$$

$$= \frac{2}{3}\cdot 2^{3/2} - \frac{2}{3}\cdot 1^{3/2} = \boxed{\frac{2}{3}(2\sqrt{2}-1)}$$

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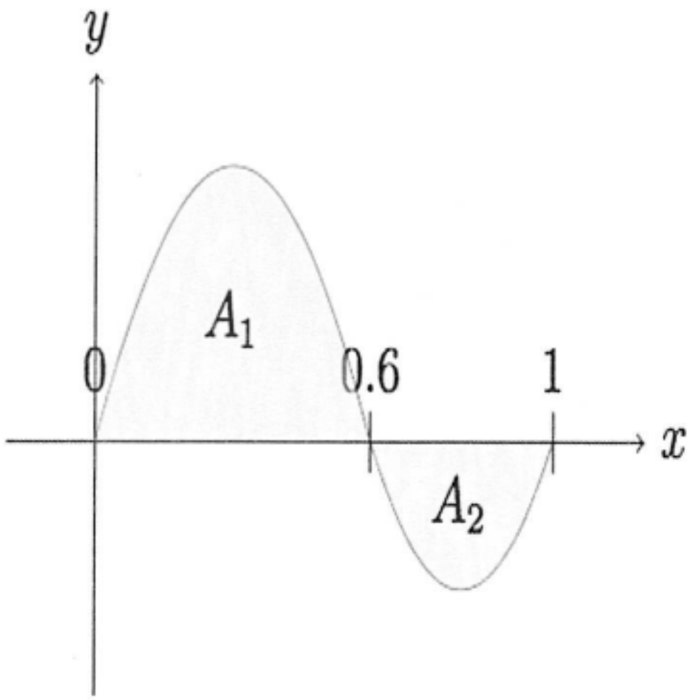
3. (10 points) The graph of a function $y = f(x)$ is shown below. Let $g(x) = \int_0^x f(t)dt$ on the interval $[0, 1]$. The areas $A_1 = 5$, $A_2 = 4$.



(a) (2 points) Evaluate $g(1)$.

$$g(1) = \int_0^1 f(t)\,dt = A_1 - A_2 = 5 - 4 = \boxed{1.}$$

negative, since below axis

(b) (2 points) Find $g'(0.6)$.

$$g'(x) = \frac{d}{dx}\int_0^x f(t)dt = f(x), \quad \text{so } g'(0.6) = f(0.6) = \boxed{0,} \text{ from the graph.}$$

(c) (6 points) On which interval(s) is $g(x)$ increasing?decreasing?

It increases when $f(x)$ is positive: $0 < x < 0.6$

decreases when $f(x)$ is negative: $0.6 < x < 1$.

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4. (16 points) Use partial fractions to evaluate the integral:

$$\int \frac{(4x^2 - 12x + 12)\,dx}{x^3 - 4x^2 + 4x}$$

(a) (5 points) Set up the partial fraction decomposition for this function. Use A, B, and C as your coefficients.

Factoring, we get $x^3-4x^2+4x = x(x^2-4x+4) = x(x-2)^2$.

Repeat factor, so

$$\frac{4x^2-12x+12}{x^3-4x^2+4x} = \boxed{\frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}}$$

(b) (6 points) Solve for the coefficients A, B, and C in the partial fraction decomposition.

Clear denominators: $4x^2-12x+12 = (x-2)^2A + (x-2)xB + xC$.

Plug in $x=2$: $4 = 0\cdot A + 0\cdot B + 2C \Rightarrow \boxed{C=2}$

Plug in $x=0$: $12 = (0-2)^2A + 0\cdot B + 0\cdot C$

$12 = 4A \Rightarrow \boxed{A=3}$

$= x^2A - 2xA + 4A + x^2B - 2xB + xC$

x^2's on left: 4

x^2's on right: $A+B$

$A+B=4 \Longrightarrow \boxed{B=1}$

(c) (5 points) Solve the integral. If you were unable to solve for the coefficients A, B, and C, just leave them as constants in your final answer!

$$\int \frac{4x^2-12x+12}{x^3-4x^2+4x}\,dx = \int \frac{3}{x} + \frac{1}{x-2} + \frac{2}{(x-2)^2}\,dx = \boxed{3\ln|x| + \ln|x-2| - \frac{2}{x-2} + C}$$

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5. (16 points) Evaluate the following integrals

(a) (8 points) $\int_0^{\pi/2} \sin x \cos^2 x \, dx$

Substitute $u = \cos x$, so $du = -\sin x \, dx$. The integral becomes

$$\int_{u(0)}^{u(\pi/2)} -u^2 \, du = \int_1^0 -u^2 \, du = \int_0^1 u^2 \, du = \frac{u^3}{3}\Big|_0^1 = \boxed{\frac{1}{3}}$$

(b) (8 points) $\int x \ln x \, dx$

Integrate by parts. $u = \ln x$, $dv = x \, dx$, so $du = \frac{1}{x} dx$, $v = \frac{x^2}{2}$.

Then

$$\int x \ln x \, dx = \frac{x^2 \ln x}{2} - \int \frac{x^2}{2} \frac{1}{x} dx = \frac{x^2 \ln x}{2} - \int \frac{1}{2} x \, dx$$

$$= \boxed{\frac{x^2 \ln x}{2} - \frac{x^2}{4} + C.}$$

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6. (16 points) Evaluate the following improper integrals.

(a) (8 points) $\int_0^\infty e^{-7x}\,dx$

A bound is infinite, so take the limit:

$$\int_0^\infty e^{-7x}dx = \lim_{L\to\infty} \int_0^L e^{-7x}\,dx = \lim_{L\to\infty} -\frac{1}{7}e^{-7x}\Big|_0^L = \lim_{L\to\infty} -\frac{1}{7}e^{-7L} + \frac{1}{7}e^0$$

$$= \lim_{L\to\infty} \frac{1}{7} - \frac{1}{7}e^{-7L} = \boxed{\frac{1}{7}}$$

goes to 0 when L large

(b) (8 points) $\int_0^1 \frac{2}{x-1}\,dx$

Function is infinite at $x=1$, so we use

$$\lim_{L\to 1^-} \int_0^L \frac{2}{x-1}dx = \lim_{L\to 1^-} 2\ln|x-1|\Big|_0^L = \lim_{L\to 1^-} \ln|L-1| - \ln|1|$$

$$= \lim_{L\to 1^-} \ln|L-1|$$

very large negative number as L goes to 1

no limit!

$\boxed{\text{divergos}}$

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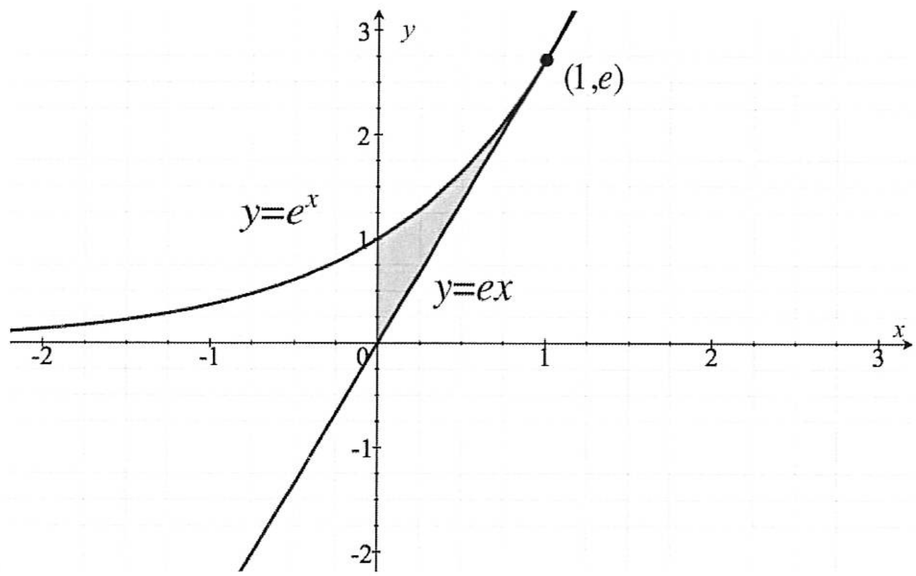
7. (12 points) Let R be the region bounded by the graphs of $y = e^x$, $y = ex$ and the y-axis.



Set up but do not compute an integral expression for the

(a) (6 points) area of R,

$$\int_0^1 e^x - ex \, dx$$

upper fct

lower fct

(b) (6 points) volume of the solid of revolution obtained by rotating R about the x-axis.

washer method gives

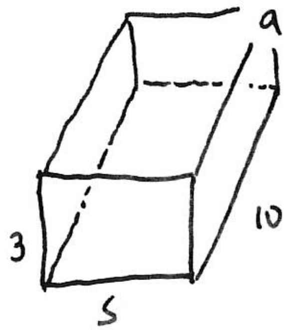
$$\int_0^1 \pi \left((e^x)^2 - (ex)^2 \right) dx.$$

8. (10 points) A rectangular tank has a rectangular base with side lengths 10 m and 5 m. The tank is 3 m high and full of water. Set up the integral for the work required to pump $\frac{1}{3}$ of the water over the top of the tank. Don't evaluate the integral. (Use $g = 9.8\ m/s^2$ and the mass density of water is $\rho_w = 1000\ kg/m^3$.)

area of water at height y

distance height y water has to move.

$$\text{work} = \int_a^b \rho g\, A(y)\, D(y)\, dy$$



we want to remove top 1/3rd of water, which is $y=2$ to $y=3$.

water at height y must be moved distance $3-y$

area $= 5 \cdot 10 = 50$, doesn't depend on y.

$$\Rightarrow \int_2^3 \rho g (50)(3-y)\, dy = \boxed{\int_2^3 (1000)(9.8)(50)(3-y)\, dy}$$

(in Joules)

9. (0 points) [The answer to this question does not in any way affect your score. Do not complete this question until after you are finished with all of the problems.] If you were to assign a grade (A, B, C, D, or F) to how you feel you did on this exam, what grade would it be? Please be honest.