

DO NOT WRITE ABOVE THIS LINE!!

MATH 181 Midterm 1

October 3, 2018

Directions. Fill in each of the boxes below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Answers

- Circle your instructor's name: Simpson Diep Shulman
- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- You must show all of your work.
- A solution for one problem may not go on another page.

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1. (10 points) Find the integral $\int \left(6x^2 - \frac{5}{x} + e^{-2x} \right) dx$.

$$= 2x^3 - 5\ln|x| - \frac{1}{2}e^{-2x} + C$$

2. (10 points) Calculate $\int \frac{4}{\sqrt{x+9}} dx$.

$$\boxed{\begin{array}{l} u = x+9 \\ du = dx \end{array}} \quad u\text{-substitution}$$

$$= \int \frac{4 du}{\sqrt{u}}$$

$$= \int 4 u^{-1/2} du$$

$$= 8 u^{1/2} + C$$

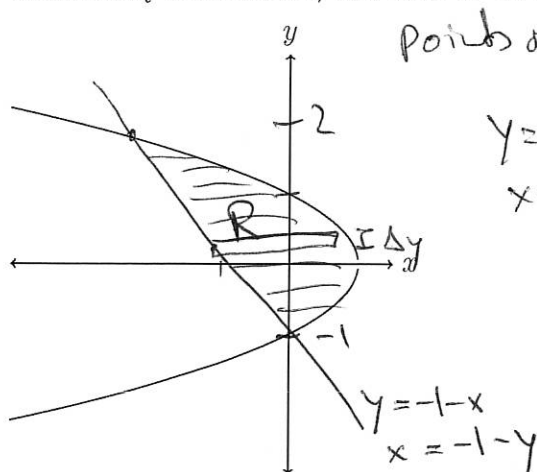
$$= 8(x+9)^{1/2} + C$$

$$= 8\sqrt{x+9} + C$$

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3. (12 points) Consider the curves $y = -1 - x$ and $x = 1 - y^2$.

(a) The curve $x = 1 - y^2$ has been graphed for you below. Draw the linear function on the same set of axes. On your graph, indicate the points of intersection of the two curves. Shade the region bounded by both curves, and label it "R".



points of intersection:

$$\begin{aligned} y &= -1 - x \\ x &= 1 - y^2 \end{aligned} \Rightarrow$$

$$y = -1 - (1 - y^2)$$

$$0 = y^2 - y - 2$$

$$0 = (y+1)(y-2)$$

$$y = -1 \text{ and } y = 2$$

- (b) Calculate the area of R, the bounded area between the two curves.

$$\text{Area} = \int_{-1}^2 \underbrace{(1 - y^2)}_{\text{right}} - \underbrace{(-1 - y)}_{\text{left}} dy$$

$$= \int_{-1}^2 (2 - y^2 + y) dy$$

$$= 2y - \frac{y^3}{3} + \frac{y^2}{2} \Big|_{-1}^2$$

$$= \left(2 \cdot 2 - \frac{8}{3} + \frac{4}{2} \right) - \left(-2 + \frac{1}{3} + \frac{1}{2} \right)$$

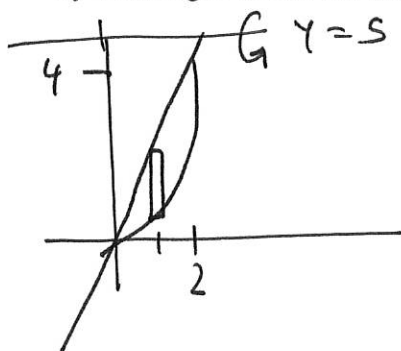
$$= \left(4 - \frac{8}{3} \right) - \left(-2 + \frac{2+3}{6} \right)$$

$$= 8 - \frac{8}{3} - \frac{5}{6}$$

$$= \frac{8 \cdot 6 - 16 - 5}{6} = \frac{27}{6} = \frac{9}{2} = \boxed{4.5}$$

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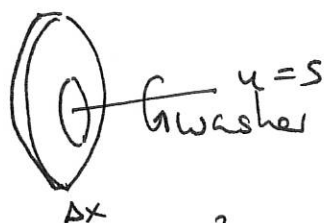
4. (12 points) Let S be the region bounded by $y = x^2$ and $y = 2x$. Find the volume of the solid obtained by rotating S about the horizontal line $y = 5$.



points of intersection

$$\begin{aligned} x^2 &= 2x \\ x^2 - 2x &= 0 \\ x(x - 2) &= 0 \\ x &= 0 \text{ or } x = 2 \end{aligned}$$

typical slice



$$\begin{aligned} R &= 5 - x^2 \\ r &= 5 - 2x \end{aligned}$$

$$0 \leq x \leq 2$$

$$\begin{aligned} \text{Volume} &= \int_0^2 \pi \cdot \left((5 - x^2)^2 - (5 - 2x)^2 \right) dx \\ &= \pi \int_0^2 \left((25 - 10x^2 + x^4) - (25 - 20x + 4x^2) \right) dx \\ &= \pi \int_0^2 (x^4 - 14x^2 + 20x) dx \\ &= \pi \left(\frac{x^5}{5} - \frac{14x^3}{3} + 10x^2 \right) \Big|_0^2 \\ &= \pi \left(\frac{32}{5} - \frac{14 \cdot 8}{3} + 10 \cdot 4 \right) - \pi \cdot (0 - 0 + 0) \\ &= \pi \frac{96 - 560 + 600}{15} = \pi \frac{136}{15} \end{aligned}$$

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5. (10 points) Find the average value of $f(\theta) = \sec^2 \theta \cdot \tan^2 \theta$ on the interval $\left[0, \frac{\pi}{4}\right]$. Simplify your answer.

$$\begin{aligned}\text{Ave value} &= \frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} \sec^2 \theta \tan^2 \theta \, d\theta \\ &= \frac{4}{\pi} \int_0^1 u^2 \, du\end{aligned}$$

$$\begin{aligned}u &= \tan \theta \\ du &= \sec^2 \theta \, d\theta\end{aligned}$$

u -substitution

$$\theta = 0 \Rightarrow u = \tan(0) = 0$$

$$\theta = \pi/4 \Rightarrow u = \tan(\pi/4) = 1$$

$$= \frac{4}{\pi} \cdot \frac{u^3}{3} \Big|_0^1$$

$$= \boxed{\frac{4}{3\pi}}$$

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6. (12 points) Choose ONE of the following integrals to compute. You may only choose one of them, and must CIRCLE the integral you're choosing to compute, and then compute it.

$$\int x^3 \ln x \, dx$$

by parts

$$\begin{array}{ll} u = \ln x & dv = x^3 dx \\ du = \frac{dx}{x} & v = \frac{x^4}{4} \end{array}$$

$$= \frac{x^4}{4} \cdot \ln x - \int \frac{x^3}{4} dx$$

$$= \frac{x^4}{4} \cdot \ln x - \frac{x^4}{16} + C$$

$$\int x e^{-3x} dx$$

by parts

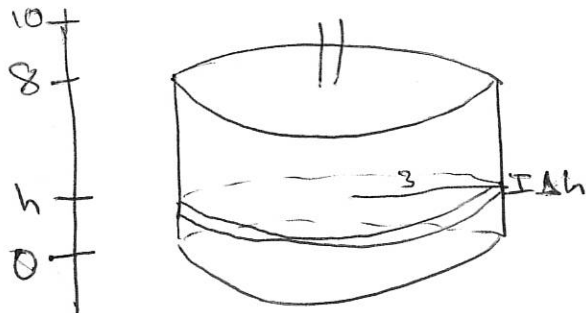
$$\begin{array}{ll} u = x & dv = e^{-3x} dx \\ du = dx & v = -\frac{1}{3} e^{-3x} \end{array}$$

$$= -\frac{1}{3} x e^{-3x} + \int \frac{1}{3} e^{-3x} dx$$

$$= -\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x} + C$$

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7. (10 points) A cylindrical tank is of height 8 meters and base radius of 3 meters, and it is standing on its circular base. It is half full of oil, and we want to pump it all out by a pipe that extends 2 meters above the top of the tank. Assume that the density of oil is $\rho = 9000 \frac{N}{m^3}$ and the acceleration due to gravity is $g = 9.8 \frac{m}{s^2}$. Set up the integral for the work to pump all the oil out of the tank, again assuming the tank is only half full. DO NOT EVALUATE THE INTEGRAL!



$$\text{force of slice} = (\pi \Delta h) \cdot (9000 \cdot 9.8)$$

$$\text{distance} = 10 - h$$

$$0 \leq h \leq 4$$

$$\Rightarrow \text{Work} = \int_0^4 (10 - h) (\pi) (9000 \cdot 9.8) dh \text{ Joules}$$

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8. (12 points) Consider the function $f(x) = \frac{3}{x(x^2+1)}$.

(a) Find the partial fraction decomposition of f .

$$\frac{3}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$3 = A(x^2+1) + (Bx+C)x$$

$$A=3$$

$$x=0 \quad 3 = A$$

$$= 3(1+1) + (B+C)$$

$$x=1 \quad 3 = 6 + B+C \Rightarrow B+C = -3$$

$$x=-1 \quad 3 = 3(1+1) + (-B+C)(-1)$$

$$3 = 6 + B - C \Rightarrow B - C = -3$$

$$B+C = -3$$

$$B-C = -3$$

$$2B = -6$$

$$B = -3 \Rightarrow C = 0$$

$$\boxed{\frac{3}{x(x^2+1)} = \frac{3}{x} + \frac{-3x}{x^2+1}}$$

(b) Find $\int f(x) dx$.

$$= \int \frac{3}{x} dx - \int \frac{3x}{x^2+1} dx$$

$$\int \frac{3x}{x^2+1} dx = \frac{3}{2} \int \frac{du}{u} = \frac{3}{2} \ln|u| + c$$

$$= \frac{3}{2} \ln|x^2+1| + c$$

$$\boxed{u = x^2+1}$$

$$\boxed{du = 2x dx}$$

$$\boxed{= 3 \ln|x| - \frac{3}{2} \ln|x^2+1| + c}$$

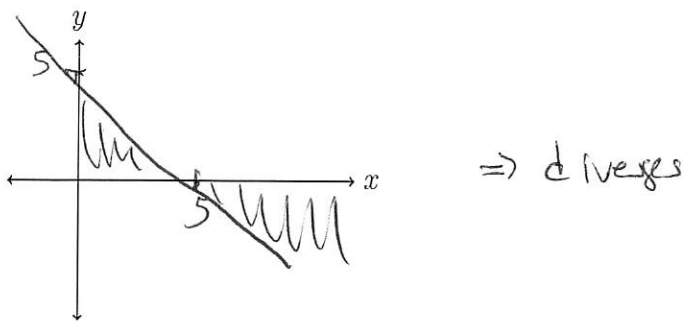
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9. (12 points) Consider the integral $\int_0^{\infty} (5-x) dx$.

(a) Write this improper integral in terms of a limit of a proper integral.

$$= \lim_{b \rightarrow \infty} \int_0^b (5-x) dx$$

(b) Sketch a graph of the function and shade the part that represents the value of the integral.



(c) Find the value of the integral if it converges, or show that it diverges by using your answer from part (a). [You should also check your answer with part (b) to see if it makes sense.]

$$\begin{aligned} \lim_{b \rightarrow \infty} \int_0^b (5-x) dx &= \lim_{b \rightarrow \infty} \left(5x - \frac{x^2}{2} \right) \Big|_0^b \\ &= \lim_{b \rightarrow \infty} \left(5b - \frac{b^2}{2} \right) - 0 = -\infty \end{aligned}$$

diverges

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PAPER, BUT NOTHING ON THIS PAGE WILL BE GRADED.