MATH 181 Midterm 1

October 2, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 10 pages (including this cover page) and 8 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

Name:_____

UIN:_____

UIC Email:_____

Signature: _____

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

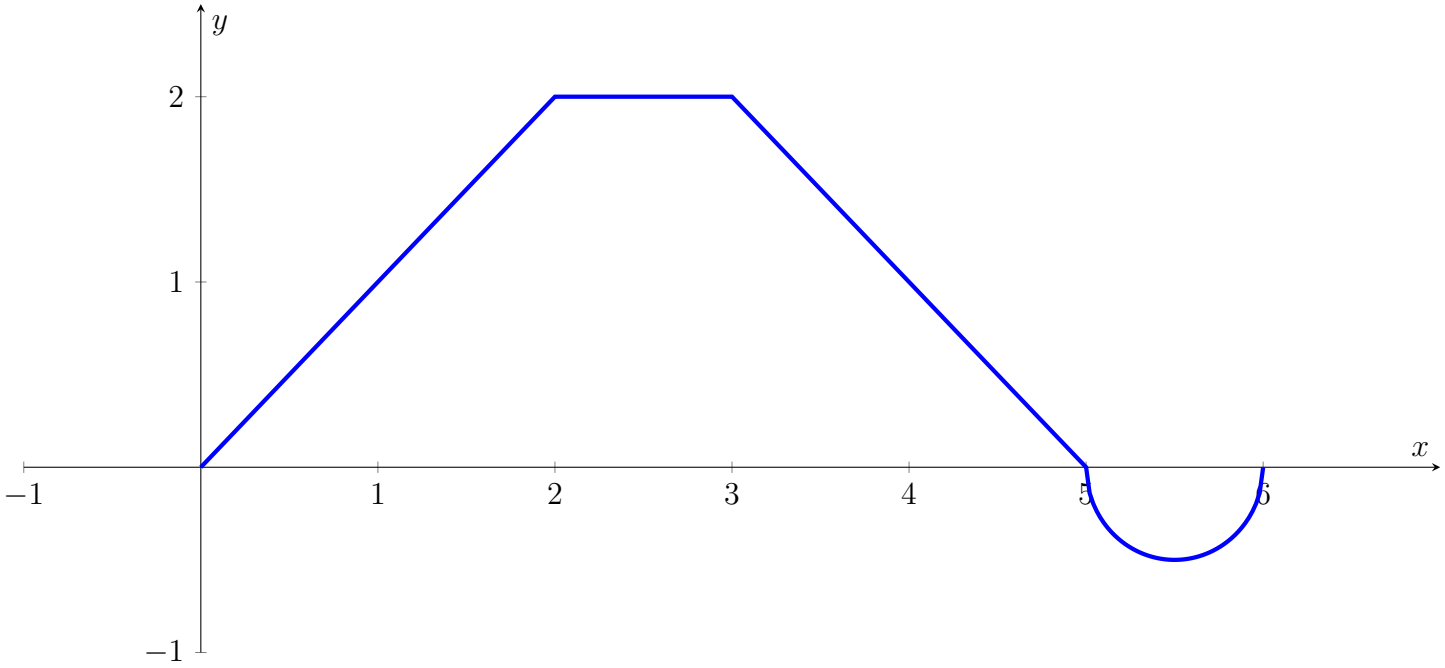
- Drew Shulman
- Mercer (Tabes) Bridges
- Nick Switala

Write your Initials (First, Middle, Last)_____

1. (12 points) Answer the following by circling the correct answer(s). You do not need to show any work.

   (a) (4 points) Below the possible answers is the graph of a piecewise-smooth function f whose pieces are all straight lines or semicircles. What is $\int_0^6 f(x)\,dx$?

   A) $6 + \frac{\pi}{8}$ **B)** $6 - \frac{\pi}{8}$ **C)** $6 + \frac{\pi}{4}$ **D)** $6 - \frac{\pi}{4}$



   (b) (8 points) Which of the following definite integrals evaluate to 0? (Circle ALL correct answers)

   A) $\int_{-\pi/4}^{\pi/4} \cos^3\theta\, d\theta$

   B) $\int_{-3\pi}^{3\pi} x^3 + \sin(x)\, dx$

   C) $\int_0^{2\pi} \cos(x)\, dx$

   D) $\int_{-2}^{2} |x| - 1\, dx$

2. (24 points) Compute the following integrals:

(a) (8 points) $\int x \cos(3x)\, dx$

(b) (8 points) $\int x\sqrt{x-5}\, dx$

(c) (8 points) $\int \sin^3(x)\cos^2(x)\, dx$

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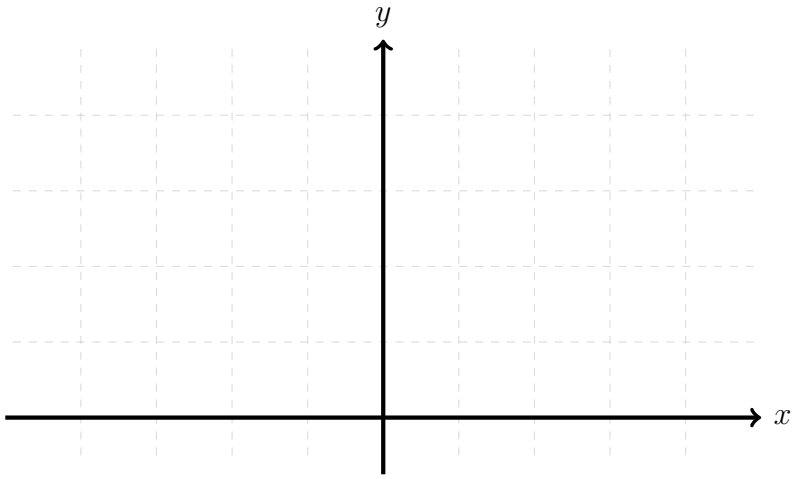
3. (19 points) Consider the region R bounded between $y = 4 - x^2$ and $y = x + 2$.

   (a) (4 points) Sketch R on the axes below, making sure to determine and label the intersection points of the two curves.



   (b) (8 points) Compute the area of R.

   (c) (7 points) Set up, but DO NOT EVALUATE, an integral to compute the volume of the solid formed by revolving R around the x-axis.

Write your Initials (First, Middle, Last)_____

4. (12 points) Suppose $\int_0^4 f(x)\,dx = 6$. Compute the following integrals by performing a substitution:

   (a) (6 points) $\int_0^2 xf(x^2)\,dx$

   (b) (6 points) $\int_{-2}^2 2f(x+2)\,dx$

Write your Initials (First, Middle, Last)_____

5. (9 points) Consider a circular swimming pool of radius 6 ft and height 5 ft, and suppose the pool contains 4 feet of water. Assuming the density of water is 1 g/cm^3 and that the acceleration due to gravity is 10 m/s^2, set up but DO NOT EVALUATE an integral to compute the work needed to pump all of the water out over the side of the pool.

Write your Initials (First, Middle, Last)_____

6. (15 points) In this problem you will compute $\int \frac{2x-3}{x^2(x-1)}\,dx$.

   (a) (3 points) Find the general form of the partial fraction decomposition for $f(x) = \frac{2x-3}{x^2(x-1)}$ using coefficients $A, B, ...$

   (b) (6 points) Solve for $A, B, ...$ in the partial fraction decomposition.

   (c) (6 points) Compute the integral. If you could not finish the previous part, just write your answer in terms of the unknowns $A, B, ...$

Write your Initials (First, Middle, Last)_____

7. (14 points) In this question, you will investigate the length of the curve $y = \ln(\cos(x))$.

   (a) (8 points) Set up an integral which computes the length of the curve over the interval $[0, \pi/4]$.

   (b) (6 points) Evaluate the integral from the previous part. The following formulae may be useful:

$$\int \tan x \; dx = -\ln|\cos x| + C \qquad \int \sec x \; dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x \; dx = \ln|\sin x| + C \qquad \int \csc x \; dx = -\ln|\csc x - \cot x| + C$$

Write your Initials (First, Middle, Last)_____

8. (16 points) Compute the following improper integrals (if they exist); in each case, you should first express the improper integral as a limit of proper integrals, and check whether the integral converges or not.

   (a) (8 points) $\int_1^\infty \frac{3}{x^3}\,dx$

   (b) (8 points) $\int_0^1 \frac{3}{x^3}\,dx$

This page can be used as scratch paper. It WILL NOT BE GRADED, so please SHOW YOUR WORK WITH YOUR PROBLEMS.