DO NOT WRITE ABOVE THIS LINE!!

MATH 181 Exam 1
February 15, 2017

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: ______________________________

University Email: ______________________________

UIN: ______________________________

Circle your instructor's name: Boester Cappetta Steenbergen

TA's Name: ______________________________

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. (14 points) Evaluate the following integrals.

(a) $\int_1^e \frac{\ln x}{x}\, dx$ by substitution

$\boxed{\begin{array}{l} u = \ln x \\ du = \frac{dx}{x} \end{array}}$

$\left.\begin{array}{l} x = 1 \Rightarrow u = \ln 1 = 0 \\ x = e \Rightarrow u = \ln e = 1 \end{array}\right\}$ new limits of integration

$= \int_0^1 u\, du$

$= \frac{1}{2} u^2 \Big|_0^1 = \frac{1}{2}(1)^2 - \frac{1}{2}(0)^2 = \boxed{\frac{1}{2}}$

(b) $\int x^2 \cos(3x)\, dx$

by parts

$u = x^2 \qquad dv = \cos(3x)\, dx$

$du = 2x\, dx \qquad v = \frac{1}{3}\sin(3x)$

$= \frac{1}{3}x^2 \sin(3x) - \int \frac{2}{3} x \sin(3x)\, dx$

by parts again

$u = \frac{2}{3}x \qquad dv = \sin(3x)\, dx$

$du = \frac{2}{3}\, dx \qquad v = -\frac{1}{3}\cos(3x)$

$= \frac{1}{3}x^2 \sin(3x) - \left(-\frac{2}{9} x \cos(3x) + \frac{2}{9}\int \cos(3x)\, dx\right)$

$= \boxed{\frac{1}{3}x^2 \sin(3x) + \frac{2}{9} x \cos(3x) - \frac{2}{27}\sin(3x) + c}$

2. (14 points) Use partial fractions to evaluate the integral:

$$I = \int \frac{7x-3}{x^3-3x^2}\,dx$$

(a) Set up the partial fraction decomposition for this function using coefficients A, ...

$$\frac{7x-3}{x^3-3x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$$

(b) Solve for the coefficients A, ... in the partial fraction decomposition.

$$7x-3 = Ax(x-3) + B(x-3) + Cx^2$$

$x=0$ $\quad -3 = B(-3) \Rightarrow \boxed{B=1}$

$x=3$ $\quad 18 = 9C \Rightarrow \boxed{C=2}$

$x=1$ $\quad 4 = A(1)(-2) + \cancel{1\cdot(-2)} + 2\cancel{(1)^2} \Rightarrow \boxed{A=-2}$

(c) Solve the integral using the coefficients you found above. If you were unable to find values for A, ... , leave them as unknown constants in your answer.

$$\Rightarrow I = \int \frac{-2}{x}\,dx + \int \frac{dx}{x^2} + \int \frac{2\,dx}{x-3}$$

$$= \boxed{-2\ln|x| - \frac{1}{x} + 2\ln|x-3| + c}$$

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3. (14 points) Answer the following integral questions.

(a) Evaluate $\displaystyle\int_0^{\infty} e^{-3x}\,dx$

$$= \lim_{b\to\infty} \int_0^b e^{-3x}\,dx$$

$$= \lim_{b\to\infty} -\frac{1}{3}e^{-3x}\Big|_0^b$$

$$= \lim_{b\to\infty} -\frac{1}{3}e^{-3b} + \frac{1}{3}e^0$$

$$= 0 + \frac{1}{3}$$

$$= \boxed{\frac{1}{3}}$$

(b) DO NOT SOLVE. Use limits to properly restate the integral $\displaystyle\int_{-\infty}^{\infty} \frac{\arctan(x)}{x}\,dx$

Note that this function is undefined at $x=0$!

$$= \int_{-\infty}^{0} \frac{\arctan x}{x}\,dx + \int_0^{\infty} \frac{\arctan x}{x} \qquad f(x) = \frac{\arctan x}{x}$$

$$= \int_{-\infty}^{-1} f(x)\,dx + \int_{-1}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^{\infty} f(x)\,dx$$

$$= \lim_{a\to-\infty} \int_a^{-1} f(x)\,dx + \lim_{b\to 0^-} \int_{-1}^{b} f(x)\,dx + \lim_{c\to 0^+} \int_c^1 f(x)\,dx + \lim_{d\to\infty} \int_1^d f(x)\,dx$$

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4. (14 points) Answer the following area questions.

(a) Use the trapezoid rule with $n = 4$ to approximate $\int_0^\pi \sin x \, dx$. You do not need to simplify your answer.

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\sin x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0

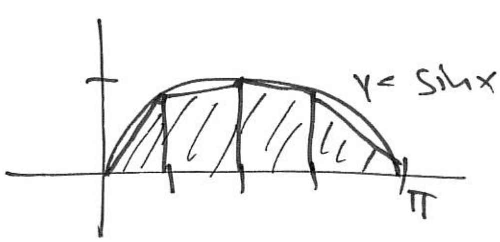
$\Delta x = \pi/4$

$$T_4 = \frac{\pi}{8} \cdot \left(0 + 2 \cdot \frac{\sqrt{2}}{2} + 2 \cdot 1 + 2 \cdot \frac{\sqrt{2}}{2} + 0\right) = \frac{\pi}{8}\left(2\sqrt{2} + 2\right)$$

$\frac{\Delta x}{2}$

(b) Is your above approximation an underestimate or overestimate of the actual area under the curve? Explain how you know.

it is an underestimate



The trapezoids are missing area from under the curve y=sin x.

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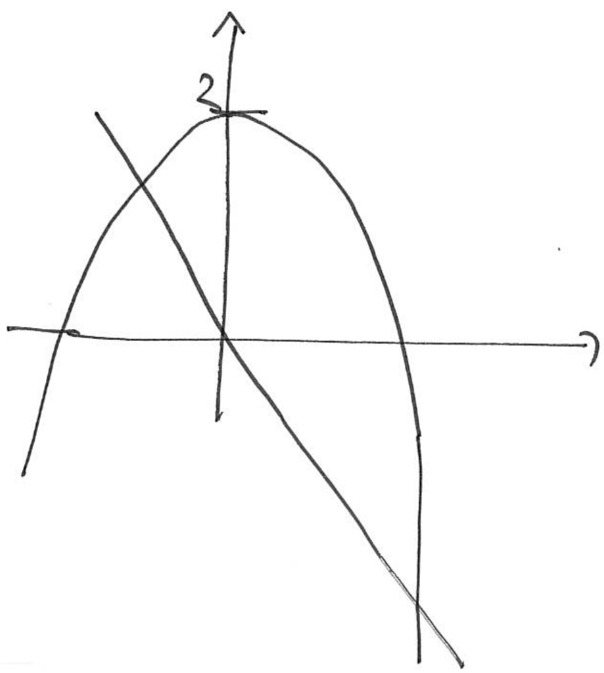
5. (12 points) Find the area of the region bounded by $y = 2 - x^2$ and $y = -x$. Be sure to sketch a graph of both functions and the enclosed region.



find points of intersection:

$2 - x^2 = -x$

$0 = x^2 - x - 2$

$0 = (x+1)(x-2)$

$x = -1$ & $x = 2$

$$\Rightarrow \text{Area} = \int_{-1}^{2} (2 - x^2) - (-x)\, dx$$

$$= 2x - \frac{x^3}{3} + \frac{x^2}{2} \Big|_{-1}^{2}$$

$$= \left(4 - \frac{8}{3} + \frac{4}{2}\right) - \left(-2 - \frac{1}{3} + \frac{1}{2}\right)$$

$$= \boxed{\frac{9}{2}}$$

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6. (8 points) SET UP THE INTEGRAL BUT DO NOT SOLVE.

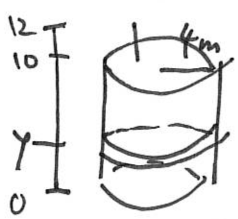
Write the integral that gives the length of the curve $f(x) = x^2 + 1$ on the interval [0,2].

$$L = \int_0^2 \sqrt{1+(2x)^2}\, dx \qquad f'(x) = 2x \rightsquigarrow 1+(f'(x))^2 = 1+(2x)^2$$

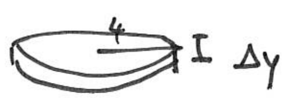
$$= \int_0^2 \sqrt{1+4x^2}\, dx$$

7. (8 points) SET UP THE INTEGRAL BUT DO NOT SOLVE.

A cylindrical tank is 10 meters tall with a 4 meter radius. It is half-full of water. If the density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.8 m/s^2, write the integral that finds the work necessary to pump the water out of the tank through an outflow pipe extending 2 meters above the top of the tank.



$\rightsquigarrow$ typical slice



force of a slice $= 1000\ kg/m^3 \cdot 9.8\ m/s^2 \cdot 16\pi\ m^2\ \Delta y\ m$

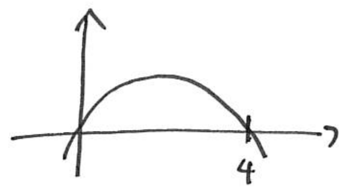
$0 \le y \le 5$

distance of slice $= 12 - y\ m$

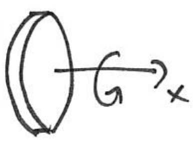
$\Rightarrow$ Work done $= \int_0^5 9800(16\pi)(12-y)\, dy$ Joules

8. (16 points) SET UP EACH INTEGRAL BUT DO NOT SOLVE.

Write the integral that gives the volume of the region bounded by the x-axis and the parabola $y = 4x - x^2$ rotated around each of the following axes:



(a) x-axis

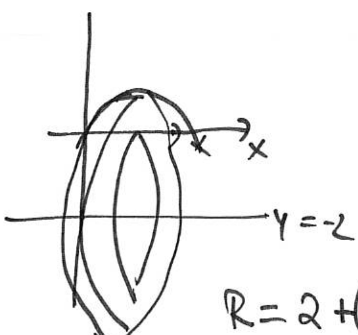


disk

$r = 4x - x^2$

$$\int_0^4 \pi (4x - x^2)^2 \, dx$$

(b) $y = -2$



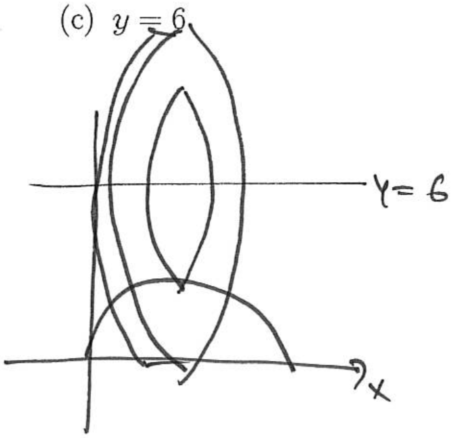
washer

$R = 2 + (4x - x^2)$

$r = 2$

$$\int_0^4 \pi \cdot \left((2 + (4x - x^2))^2 - 2^2 \right) dx$$

(c) $y = 6$



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$R = 6$

$r = 6 - (4x - x^2)$

$$\int_0^4 \pi \cdot \left(6^2 - (6 - (4x - x^2))^2 \right) dx$$