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MATH 181 Midterm 1
February 21, 2018

Directions. Fill in each of the boxes below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Answers

- Circle your instructor: Cappetta Diep Shulman
- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- You must show all of your work.
- A solution for one problem may not go on another page.

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1. (10 points) Find the average value of $f(x) = x^2 + 5$ on the interval $[0, 3]$.

$$\text{ave value} = \frac{1}{3-0}\int_0^3 (x^2+5)\,dx$$

$$= \frac{1}{3}\left(\frac{x^3}{3} + 5x\right)\Big|_0^3$$

$$= \frac{1}{3}(9+15) - \frac{1}{3}(0+0)$$

$$= 8$$

2. (10 points) Calculate $\int_{-\pi}^{\pi} (x - \sin x)\,dx$.

(A) $f(x) = x - \sin x$ is an odd function,

thus $\int_{-\pi}^{\pi} f(x)\,dx = \boxed{0}$

or

(B) $$= \frac{x^2}{2} + \cos x \Big|_{-\pi}^{\pi}$$

$$= \left(\frac{\pi^2}{2} + \underbrace{\cos\pi}_{-1}\right) - \left(\frac{(-\pi)^2}{2} + \underbrace{\cos(-\pi)}_{-1}\right)$$

$$= \frac{\pi^2}{2} - \frac{\pi^2}{2} + (-1) - (-1) = \boxed{0}$$

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3. (20 points)

(a) Find $\int \frac{dx}{\sqrt{3x+1}}$.

u-substitution

$u = 3x+1$

$du = 3dx$

$= \int \frac{1}{3} u^{-1/2} \, du$

$= \frac{2}{3} u^{1/2} + C = \frac{2}{3}\sqrt{3x+1} + C$

(b) Show that $\int_5^8 \frac{x}{x-4}\, dx = 3 + 4\ln(4)$.

$u = x-4$

$du = dx$

$\Rightarrow x = u+4$

u-substitution

$x = 5 \Rightarrow u = 5-4 = 1$

$x = 8 \Rightarrow u = 8-4 = 4$

$= \int_1^4 \frac{u+4}{u}\, du$

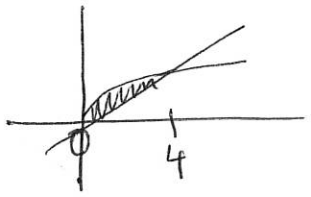
$= \int_1^4 \left(1 + \frac{4}{u}\right) du$

$= u + 4\ln|u| \Big|_1^4 = (4 + 4\ln 4) - (1 + \underbrace{4\ln 1}_{0}) = 3 + 4\ln 4$

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4. (15 points) Consider the region R bounded by $y = \sqrt{x}$ and $y = \frac{1}{2}x$.

(a) Sketch a graph of R and label the intersection points.



$\sqrt{x} = \frac{1}{2}x \Rightarrow x = \frac{1}{4}x^2 \Rightarrow x - \frac{1}{4}x^2 = 0$

$\Rightarrow x\left(1 - \frac{1}{4}x\right) = 0$

$x = 0$ or $x = 4$

(b) Find the area of R.

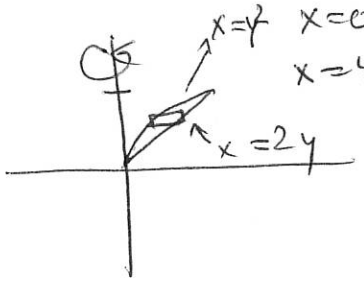
$$\int_0^4 \left(\sqrt{x} - \frac{1}{2}x\right) dx$$

$$= \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Big|_0^4$$

$$= \left(\frac{2}{3}\cdot 8 - \frac{16}{4}\right) - (0-0)$$

$$= \frac{16-12}{3} = \boxed{\frac{4}{3}}$$

(c) Set up an integral that represents the volume obtained by rotating the region R about the y-axis. Do not calculate the integral.



$x = 0 \Rightarrow y = 0$

$x = 4 \Rightarrow y = 2$

$$\int_0^2 \pi \cdot (R^2 - r^2)\, dy$$

$$= \int_0^2 \pi \left((2y)^2 - (y^2)^2\right) dy$$

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5. (10 points) Find $\int \sin^3(4x)\,dx$.

$$= \int \sin^2(4x)\cdot\sin(4x)\,dx = \int \left(1-\cos^2(4x)\right)\sin(4x)\,dx$$

$u = \cos(4x)$

$du = -4\sin(4x)\,dx$

u-substitution

$$= -\frac{1}{4}\int (1-u^2)\,du$$

$$= -\frac{1}{4}\left(u - \frac{u^3}{3}\right) + c$$

$$= -\frac{1}{4}\left(\cos(4x) - \frac{(\cos(4x))^3}{3}\right) + c$$

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6. (15 points)

(a) Find

$$I = \int \frac{x-5}{x^2+2x-3}\,dx$$

$x^2+2x-3 = (x+3)(x-1)$

use partial fractions:

$$\frac{x-5}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$x-5 = A(x-1) + B(x+3)$$

$x=1$: $1-5 = 0 + 4B \Rightarrow B = -1$

$x=-3$: $-3-5 = A(-4) + 0 \Rightarrow A = 2$

$$\Rightarrow I = \int \frac{2\,dx}{x+3} - \int \frac{dx}{x-1}$$

$$= 2\ln|x+3| - \ln|x-1| + C$$

(b) Set up the general form of the partial fraction decomposition for

$$\frac{3x-11}{(x+2)^2(x^2+3)}$$

using the variables A, B, ... You do not have to find the coefficients.

$$= \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx+D}{x^2+3}$$

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7. (10 points) Find $\int x^2 e^x \, dx$.

$u = x^2 \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x$

by parts

$= x^2 e^x - \int 2x e^x \, dx$

$u = 2x \quad dv = e^x dx$

$du = 2\,dx \quad v = e^x$

by parts again

$= x^2 e^x - \left(2x e^x - \int 2 e^x \, dx \right)$

$= x^2 e^x - 2x e^x + 2e^x + c$

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8. (10 points) Find $\int (2x-1)\ln x\, dx$.

$u = \ln x \qquad dv = (2x-1)\,dx$

$du = \frac{dx}{x} \qquad v = x^2 - x$

by parts

$= (x^2-x)\cdot \ln x - \int (x-1)\,dx$

$= (x^2-x)\cdot \ln x - \frac{x^2}{2} + x + C$