MATH 181 Exam 1

February 20, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 8 pages (including this cover page) and 8 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

TA Name:________________________

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Martina Bode
- Robert Cappetta
- Vi Diep
- John Steenbergen

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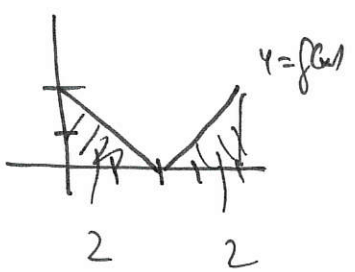
1. (4 points) Given that $\int_0^6 f(x)\,dx = 8$, $\int_4^6 2f(x)\,dx = 4$. Evaluate $\int_0^4 f(x)\,dx$.

$$\int_0^4 f(x)dx = \int_0^6 f(x)dx - \int_4^6 f(x)dx$$

$$= 8 - \frac{1}{2}\cdot 4$$

$$= 8 - 2 = \boxed{6}$$

2. (5 points) Evaluate $\int_0^4 (|x-2|)dx$ (Hint: sketch the graph of the function.)



$$\Rightarrow \int_0^4 f(x)dx = 2+2 = \boxed{4}$$

3. (10 points) Find the arc length of the function $f(x) = \frac{2}{3}x^{3/2}$ as x ranges from 0 to 3.

$$f'(x) = x^{1/2} \Rightarrow 1 + (f'(x))^2 = 1 + x$$

$$\text{arc length} = \int_0^3 \sqrt{1+x}\,dx = \int_1^4 \sqrt{u}\,du$$

u-sub $\boxed{\begin{matrix} u = 1+x \\ du = dx \end{matrix}}$

$$= \frac{2}{3}u^{3/2}\Big|_1^4$$

$$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}$$

$$= \frac{16}{3} - \frac{2}{3} = \boxed{\frac{14}{3}}$$

4. (18 points) Evaluate the following integrals:

(a) (6 points) $\int_0^{\pi} 3x \cdot \sin(x)\, dx$

by parts

$$\boxed{\begin{array}{ll} u = 3x & dv = \sin x\, dx \\ du = 3dx & v = -\cos x \end{array}}$$

$$= -3x \cos x \Big|_0^{\pi} + \int_0^{\pi} 3\cos x\, dx$$

$$= (-3\pi \cos \pi \cancel{+0\cos}(0)) + (3\sin\pi - 3\sin 0)$$

$$= \boxed{3\pi}$$

(b) (6 points) $\int x^4 \cdot \ln(x)\, dx$

by parts

$$\boxed{\begin{array}{ll} u = \ln x & dv = x^4\, dx \\ du = \frac{dx}{x} & v = \frac{x^5}{5} \end{array}}$$

$$= (\ln x)\left(\frac{x^5}{5}\right) - \int \frac{x^4}{5}\, dx$$

$$= \boxed{(\ln x)\left(\frac{x^5}{5}\right) - \frac{x^5}{25} + C}$$

(c) (6 points) $\int \cos(x)\sin^4(x)\, dx$ $= \int u^4 du = \frac{u^5}{5} + c = \frac{\sin^5 x}{5} + c$

u-sub

$$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x\, dx \end{array}}$$

5. (15 points) Use partial fractions to evaluate the integral:

$$I = \int \frac{x+7}{x(x^2-1)}\,dx$$

(a) (3 points) Set up the partial fraction decomposition for this function. Use A, B, and C as your coefficients.

$$\frac{x+7}{x(x^2-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

(b) (6 points) Solve for the coefficients A, B, and C in the partial fraction decomposition.

$$x+7 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$$

$x=0$	$7 = -A$	$\Rightarrow A=-7$
$x=1$	$8 = 2C$	$\Rightarrow C=4$
$x=-1$	$6 = 2B$	$\Rightarrow B=3$

(c) (6 points) Solve the integral. If you were unable to solve for the coefficients A, B, and C, just leave them as constants in your final answer!

$$I = \int \frac{-7}{x}\,dx + \int \frac{3}{x+1}\,dx + \int \frac{4}{x-1}\,dx$$

$$= -7\ln|x| + 3\ln|x+1| + 4\ln|x-1| + c$$

6. (16 points) Rewrite the following improper integrals as a limit and check whether or not the integrals converge or diverge. Evaluate the integral for those that converge.

(a) (8 points) $\int_0^\infty e^{-4x}\,dx$

$$= \lim_{b\to\infty} \int_0^b e^{-4x}\,dx$$

$$= \lim_{b\to\infty} -\frac{1}{4}e^{-4x}\Big|_0^b$$

$$= \lim_{b\to\infty} -\frac{1}{4}e^{-4b} + \frac{1}{4}$$

$$= 0 + \frac{1}{4}$$

$$= \boxed{\frac{1}{4}}$$

What is the limit of $\lim_{b\to\infty} e^{-4b}$?

$= 0$

(b) (8 points) $\int_0^1 \frac{2}{x-1}\,dx$

$$= \lim_{b\to 1^-} \int_0^b \frac{2}{x-1}\,dx$$

$$= \lim_{b\to 1^-} 2\ln|x-1|\Big|_0^b$$

$$= \lim_{b\to 1^-} \underbrace{2\ln|b-1|}_{-\infty} - \underbrace{2\ln|0-1|}_{=0}$$

$= -\infty \Rightarrow$ the integral diverges

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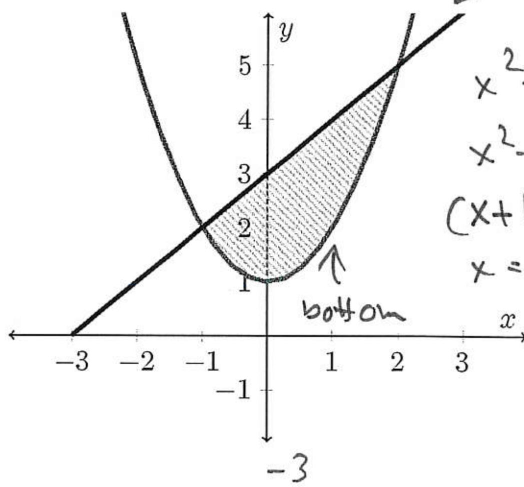
7. (18 points) Let R be the region that lies between the curves $y = x^2 + 1$ and $y = x + 3$. ↙ top $y = x+3$

(a) (8 points) Find the area of the region R.



$x^2+1 = x+3$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1$ & $x = 2$ limits of integration

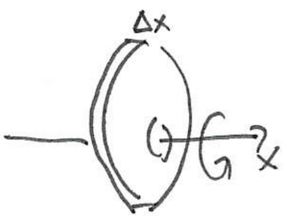
$$\text{Area} = \int_{-1}^{2} \left((x+3) - (x^2+1)\right) dx$$

$$= \int_{-1}^{2} -x^2 + x + 2 \, dx$$

$$= -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^{2}$$

$$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right) = -\frac{9}{3} + \frac{3}{2} + 6 = 3 + 1.5 = \boxed{4.5}$$

(b) (5 points) Set up the integral needed to find the volume of the solid obtained by revolving the region about the x-axis.

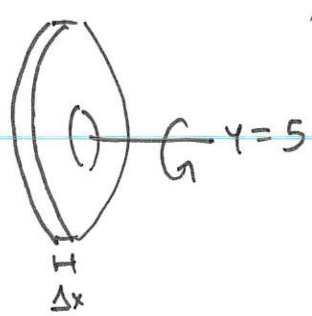


$r = x^2+1$

$R = x+3$

$$\int_{-1}^{2} \pi \cdot \left((x+3)^2 - (x^2+1)^2\right) dx$$

(c) (5 points) Set up the integral needed to find the volume of the solid obtained by revolving the region about the line $y = 5$.



$r = 5 - (x+3) = 2 - x$

$R = 5 - (x^2+1) = 4 - x^2$

$$\int_{-1}^{2} \pi \cdot \left((4-x^2)^2 - (2-x)^2\right) dx$$

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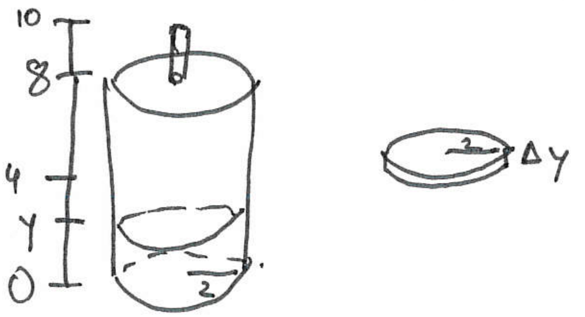
8. (8 points) A cylindrical tank is half-filled with olive oil. The radius of the tank is 2 meters and the height of the tank is 8 meters. The density of the oil is 930 kg per cubic meters and the acceleration due to gravitation is 9.8 meters per second-squared. Set up the integral for the work necessary to pump the olive oil out through an output 2 meters above the top of the tank. Do not evaluate.



force of slice $= \underbrace{(\pi \cdot 2^2 \Delta y)}_{\text{volume}} \cdot 930 \cdot 9.8$

distance $= 10 - y$

$0 \le y \le 4$

$$\Rightarrow \text{Work} = \int_0^4 (10-y) \cdot 930 \cdot 9.8 \cdot 4\pi \, dy$$

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