

MATH 181 Exam 1
February 19, 2020

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. **YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR.** This exam contains 8 pages (including this cover page) and 8 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty. Please put your initials on each page.

Your Name: _____

Your UIN: _____

Your NetID: _____

TA Name: _____

Lecture Center D4

Circle your instructor.

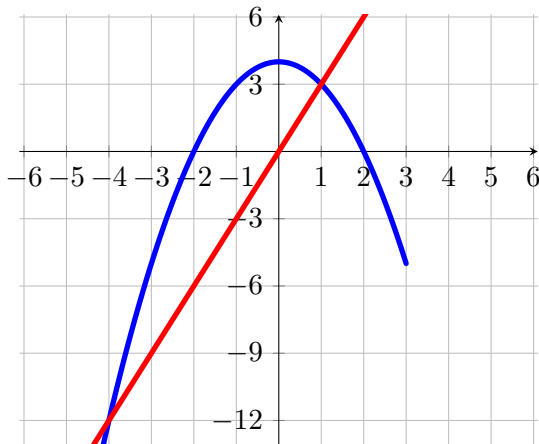
- Martina Bode
- Vi Diep
- Matthew Lee
- John Steenbergen

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

1. (8 points) **Multiple Choice.** Circle the correct answers.

(A) (4 points) Set up the integral of the area of the region bounded by $y = 4 - x^2$, and $y = 3x$, see picture below.



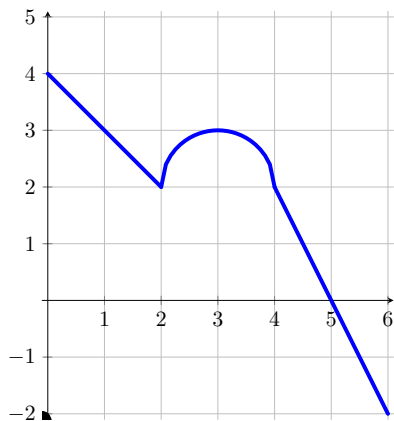
(i) $\int_{-4}^1 (4 - x^2 - 3x) dx$

(iii) $\int_{-4}^1 (3x - 4 + x^2) dx$

(ii) $\int_{-12}^3 (4 - x^2 - 3x) dx$

(iv) $\int_{-12}^3 (3x - 4 + x^2) dx$

(B) (4 points) Use the curve $y = f(x)$ provided below to evaluate the integral $\int_0^6 f(x) dx$.



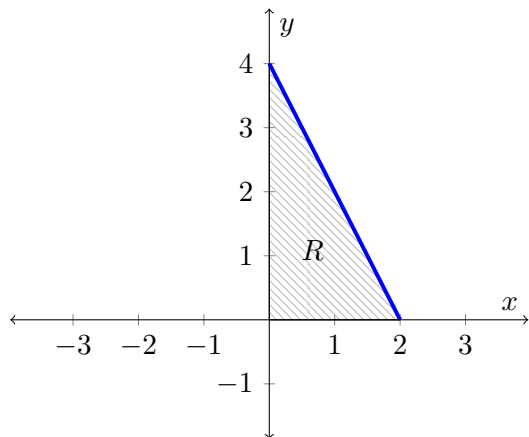
(i) $\frac{\pi}{2}$

(iii) $12 + \frac{\pi}{2}$

(ii) $10 + \frac{\pi}{2}$

(iv) $22 + \frac{\pi}{2}$

2. (12 points) Let R be the region bounded by $y = 4 - 2x$, the x - and y -axis, see picture below.



Set up the integral for the volume of the solid obtained by rotating the region R around the given line of rotation. Circle the correct answers. In each case sketch the picture of a typical slice.

(a) (6 points) about the y -axis.

(i) $\int_0^2 \pi(4 - 2x)^2 dx$

Sketch a typical slice here.

(ii) $\int_0^4 \pi(4 - 2x)^2 dx$

(iii) $\int_0^2 \pi\left(2 - \frac{1}{2}y\right)^2 dy$

(iv) $\int_0^4 \pi\left(2 - \frac{1}{2}y\right)^2 dy$

(b) (6 points) about the line $y = -1$.

(i) $\int_0^2 \pi(3 - 2x)^2 dx$

Sketch a typical slice here.

(ii) $\int_0^2 \pi\left((3 - 2x)^2 - 1\right) dx$

(iii) $\int_0^2 \pi\left((4 - 2x)^2 - 1\right) dx$

(iv) $\int_0^2 \pi\left((5 - 2x)^2 - 1\right) dx$

3. (20 points) Evaluate the following definite integrals. Suppose f is a function with the following properties:

$$\bullet \int_0^1 f(x) dx = 1$$

$$\bullet \int_0^{\pi/2} f(x) dx = 3$$

(a) (7 points) $\int_e^{e^3} \frac{\ln x}{5x} dx$

(b) (7 points) $\int_{-3}^3 (|x| + 2) dx$

(c) (6 points) $\int_0^{\pi/2} \cos x \cdot f(\sin x) dx$

4. (16 points) Evaluate the following integrals.

(a) (7 points) $\int (3x - 2)e^x dx$

(b) (9 points) $\int \sin^3 x dx$

5. (12 points) Consider the curve given by $f(x) = \frac{2}{3}(x - 1)^{3/2}$.

(a) (8 points) Set up the integral for the arc length of the curve from $(1, 0)$ to $(4, 2\sqrt{3})$.

(b) (4 points) Calculate the arc length.

6. (10 points) An empty cylindrical water tower tank has radius $7m$ and height $12m$, and the bottom of the tank is $50m$ above ground level. Assume the density of water is $1000kg/m^3$ and that gravity is $9.8m/s^2$. Set up **BUT DO NOT COMPUTE** the integral for the work done if the tank is filled to the top with water that is pumped in from ground level.

7. (10 points) Consider the integral $\int \frac{x^2 + 2}{x^3 - x^2 - 2x} dx$.

(a) (3 points) Set up the partial fraction decomposition for the rational function. Use A , B , and C as your coefficients.

(b) (3 points) Solve for the coefficients A , B , and C in the partial fraction decomposition.

(c) (4 points) Solve the integral. If you were unable to solve for the coefficients A , B , and C , just leave them as constants in your final answer!

8. (12 points) Rewrite the following improper integrals as a limit and then evaluate them, or state that they diverge.

(a) (6 points) $\int_2^3 \frac{dx}{x-2}$

(b) (6 points) $\int_4^{\infty} \frac{dx}{(x-2)^2}$