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# MATH 181 Exam 2
## October 26, 2016

Directions. Fill in each of the lines below. Circle your instructor's name and write your TA's name. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

Print Name: _____

University Email: _____

UIN: _____

Circle your instructor's name: Bode Lesieutre

TA's Name: _____

- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Make clear to the grader what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. (10 points) Find an expression for the $n^{th}$ partial sum, $S_n$, of the series

$$\sum_{k=3}^{\infty} \left( \frac{1}{k+1} - \frac{1}{k+2} \right).$$

Does this series converge or diverge? Determine its value if it converges.

2. (10 points) Let $a_n$ be a sequence of numbers such that $\frac{n}{n^3+1} \leq a_n \leq \frac{1}{n^2}$ for all $n \geq 3$. Determine whether the following series converges or diverges:

$$\sum_{n=3}^{\infty} a_n.$$

Justify your answer using appropriate tests and rules. State the name of each test used, and make sure all hypothesis of each test are satisfied.

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3. (10 points) Let $c$ be a constant, and consider the series:

$$\sum_{n=0}^{\infty} 3 \cdot \left(\frac{c}{3}\right)^n$$

**(a)**(6 points) For what values of $c$ does the series converge?

**(b)**(4 points) For those values of $c$ for which the series converges, what is the sum?

4. (12 points) Check if the following series is (a) convergent, and (b) if it is also absolutely convergent! Justify your answer!

$$\sum_{n=4}^{\infty} \frac{(-1)^n}{\sqrt{n}-1}$$

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5. (14 points) Test the following series for convergence or divergence. Justify your answer!

(a) (6 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+2}{n^3-2}$

(b) (8 points) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{n^4+2n+2}$

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6. (20 points) Find the radius and the interval of convergence for the following power series. Test the endpoints and justify your answers!

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n3^n}$$

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7. (12 points)

(a) Find the 3rd-degree Taylor polynomial of $g(x) = \dfrac{1}{x}$ about $a = 1$.

(b) Use your results from part (a) to approximate $\frac{1}{1.1}$. You do not need to simplify your answer.

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8. (12 points)

(a) Use the Maclaurin series $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots$

of $e^x$ to find the Maclaurin series of $e^{(-x^3)}$.

(b) Use the first three non-zero terms of the series you found in part (a) to approximate the definite integral.

$$\int_0^1 e^{(-x^3)}\, dx$$

You do not need to simplify your answer.