

MATH 181 Midterm 2

October 30, 2019

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR EXAM PROCTOR. This exam contains 10 pages (including this cover page) and 8 problems. After starting the exam, check to see if any pages are missing. Enter all requested information on this page. You are expected to abide by the University's rules concerning Academic Honesty.

Name: _____

UIN: _____

UIC Email: _____

Signature: _____

The following rules apply:

- You may *not* use your books, notes, calculators, or any electronic device including cell phones. Only pencils/pens allowed.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You *must* complete your work in the space provided. We will be scanning your answers into our grading system, so any work you do that is out of place, too close to the page border, or on the wrong page will *not* be graded!

Circle your instructor.

- Drew Shulman
- Mercer (Tabes) Bridges
- Nick Switala

Write your Initials (First, Middle, Last)_____

1. (20 points) Answer the following by circling the correct answer(s). You do not need to show any work.

(a) (8 points) Which of the following converge? (Circle ALL correct answers)

A) the sequence $a_n = \frac{n}{1+n^2}$

A) the sequence $a_n = \frac{(-1)^n n}{1+n^2}$

C) the series $\sum_{k=0}^{\infty} \frac{k}{1+k^2}$

D) the series $\sum_{k=0}^{\infty} \frac{(-1)^k k}{1+k^2}$

(b) (4 points) Suppose r is a real number such that $\sum_{k=0}^{\infty} 3r^k = 6$. What is r ?

A) 2

B) $1/2$

C) $3/2$

D) $5/6$

(c) (8 points) The series $\sum_{k=2}^{\infty} \frac{1}{k-1}$ is divergent. Which of the following tests could be used to show this?
(Circle ALL correct answers)

A) Comparison Test using $\sum_{k=2}^{\infty} \frac{1}{k}$

B) Limit Comparison Test using $\sum_{k=2}^{\infty} \frac{1}{k}$

C) Ratio Test

D) Divergence Test

Write your Initials (First, Middle, Last)_____

2. (32 points) Determine whether the following series converge or diverge. In each case, state which test(s) you are using and show all work!

(a) (8 points) $\sum_{k=1}^{\infty} \frac{2k^4 + k^3 + 1}{3k^4 - 4}$

(b) (8 points) $\sum_{k=1}^{\infty} \frac{k}{e^k}$

(c) (8 points) $\sum_{k=1}^{\infty} \frac{3}{2 + e^k}$

(d) (8 points) $\sum_{k=1}^{\infty} \frac{-3}{\sqrt{k}}$

Write your Initials (First, Middle, Last)_____

3. (10 points) Consider the series $\sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+3}} \right)$.

(a) (7 points) Find a formula for the n -th partial sum $\sum_{k=1}^n \left(\frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+3}} \right)$. (Hint: try writing out the first few terms until you see a pattern).

(b) (3 points) Using your answer from part (a), determine the sum of the infinite series.

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4. (12 points) Consider a function $f(x)$ with $f(5) = 1$, $f'(5) = 3$, $f''(5) = -2$, and $f'''(5) = 6$.

(a) (8 points) Write the third-order Taylor polynomial $p_3(x)$ for $f(x)$, centered at $a = 5$.

(b) (4 points) Use your answer from part (a) to estimate $f(5.1)$. You may leave your answer unsimplified.

Write your Initials (First, Middle, Last)_____

5. (16 points) Determine the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{(x-4)^k}{k3^k}$. Remember to test the endpoints.

Write your Initials (First, Middle, Last)_____

6. (16 points) (a) (7 points) Does the series $\sum_{k=1}^{\infty} \frac{7^k}{k!}$ converge?

(b) (6 points) Does the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 7^k}{k!}$ converge?

(c) (3 points) Does the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 7^k}{k!}$ converge conditionally, converge absolutely, or diverge?
Explain, making sure to indicate what tests you applied to which series.

Write your Initials (First, Middle, Last)_____

7. (14 points) Consider the power series $1 - \ln(x + 1) = 1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \cdots$.

(a) (8 points) Find a power series representation for $1 - \ln(x^3 + 1)$, making sure to write out at least 4 nonzero terms.

(b) (6 points) Write the first three non-zero terms of a power series representation for $\frac{-3x^2}{x^3 + 1}$ (hint: use part (a)).

Write your Initials (First, Middle, Last)_____

8. (10 points) Does the series $\sum_{k=1}^{\infty} \frac{(-3)^{k+1}}{4^{2k}}$ converge or diverge? If it converges, find its sum and simplify it. Show all work and state any tests that you use!

This page can be used as scratch paper. It WILL NOT BE GRADED, so please SHOW YOUR WORK WITH YOUR PROBLEMS.