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MATH 181 Midterm 2
March 21, 2018

Directions. Fill in each of the boxes below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR. Good luck!

- Circle your instructor: Cappetta Diep Shulman
- VERY IMPORTANT!!! CHECK THAT THE NUMBER AT THE TOP OF EACH PAGE OF YOUR EXAM IS THE SAME. IT IS THE NUMBER PRECEDED BY A POUND (#) SIGN. IF THEY ARE NOT ALL THE SAME, NOTIFY YOUR INSTRUCTOR OR TA RIGHT AWAY.
- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your exam will not be read and therefore not graded.
- You must show all of your work and justify all answers.
- A solution for one problem may not go on another page.
- If you are asked to calculate an improper integral, make sure you justify your answer if it converges or diverges.

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1. (8 points) Fill in each of the blanks using the answers provided. Write your answer in the blank (do not just circle it).

   (a) Given a geometric series $\sum_{k=1}^{\infty} a \cdot r^k$, this series will converge when $|r| < 1$.

   If the above series is convergent, then it will converge to $\frac{ar}{1-r}$.

   - $|r| \leq 1$
   - $|r| \geq 1$
   - $|r| < 1$
   - $|r| > 1$

   first term $= ar$

   - $\frac{a}{1-r}$
   - $\frac{ar}{1-a}$
   - $\frac{ar}{1-r}$
   - $\frac{1}{1-r}$

   (b) The Limit Comparison Test can only be used when both series $\sum a_k$ and $\sum b_k$ have positive terms.

   - positive
   - negative
   - alternating

   (c) Again using the Limit Comparison Test, if $\lim_{k\to\infty} \frac{a_k}{b_k} = L$, then we know that the series either both converge or both diverge if L satisfies $0 < L < \infty$

   - $0 < L < \infty$
   - $L = 0$
   - $0 \leq L < 1$
   - $L < 0$

2. (15 points) Consider the series

$$\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} + \cdots$$

   (a) Write this series in sigma notation by filling in below.

$$\sum_{k=0}^{\infty} (-1)^k \cdot \left(\frac{2}{3}\right)^{k+1}$$

   (b) Determine if the series converges or diverges. If it converges, find what it converges to and simplify your answer to a reduced fraction.

   geometric series with $a = \frac{2}{3}$ and $r = \frac{-2}{3}$,

   it converges to $\frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{\frac{2}{3}}{\frac{5}{3}} = \frac{2}{3} \cdot \frac{3}{5} = \boxed{2/5}$

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3. (12 points) Consider the sequence where $a_1 = \frac{2}{2}$, $a_2 = \frac{3}{5}$, $a_3 = \frac{4}{10}$, $a_4 = \frac{5}{17}$, ...

(a) Find a formula for a_n.

$$a_n = \frac{n+1}{n^2+1}$$

(b) Determine if this sequence converges or diverges. If it converges, find what it converges to.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{n^2+1} = 0$$ The sequence converges to 0.

4. (12 points) Consider a function $f(x)$ such that $f(2) = 1$, $f'(2) = 3$, $f''(2) = 9$, and $f'''(2) = 27$.

(a) Write the third order Taylor polynomial for $f(x)$ centered at $a = 2$.

$$T_3(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3$$

$$T_3(x) = 1 + 3(x-2) + \frac{9}{2}(x-2)^2 + \frac{27}{6}(x-2)^3$$

(b) Use your answer from (a) to estimate $f(1)$. Simplify your answer.

$$f(1) \approx 1 + 3(-1) + \frac{9}{2}(-1)^2 + \frac{\cancel{27}^{\,9}}{\cancel{6}_{\,2}}(-1)^3$$

$$1 - 3 + \frac{9}{2} - \frac{9}{2}$$

$$= \boxed{-2}$$

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5. (10 points) Either calculate the following improper integral, or show that it diverges.

$$\int_0^\infty x^4 e^{-x^5}\, dx$$

$$= \lim_{b\to\infty} \int_0^b \frac{x^4}{e^{x^5}}\, dx$$

u-sub $\quad u = x^5$

$du = 5x^4\, dx$

$$= \lim_{b\to\infty} \int_0^{b^5} \frac{1}{5} e^{-u}\, du$$

$$= \lim_{b\to\infty} \left(-\frac{1}{5} e^{-b^5} + \frac{1}{5} e^{0} \right)$$

$$= \qquad 0 + \frac{1}{5} \quad = \frac{1}{5}$$

$\Rightarrow \int_0^\infty x^4 e^{-x^5} dx$ converges to $\frac{1}{5}$

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6. (10 points) Determine if the following series converges or diverges. $\sum_{k=0}^{\infty} \frac{4^n}{n!}$

ratio test

$$\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left|\frac{4^{k+1}}{(k+1)!} \cdot \frac{k!}{4^k}\right|$$

$$= \lim_{k\to\infty} \frac{4}{k+1}$$

$$= 0$$

$\Rightarrow$ the series converges!

7. (10 points) Determine if the following series converges or diverges. $\sum_{k=0}^{\infty} \frac{4+k^2}{k^3+2k-7}$

limit comparison

$$a_k = \frac{4+k^2}{k^3+2k-7} \geq 0 \quad \& \quad b_k = \frac{1}{k} \geq 0$$

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{4+k^2}{k^3+2k-7} \cdot \frac{k}{1} = 1$$

$\sum \frac{1}{k}$ diverges & since $0 < 1 < \infty$ it follows
(p=1 p-test)
that both series diverge

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8. (8 points) Determine if the following series converges or diverges. $\sum_{k=1}^{\infty} \frac{k^2}{3^{k^2}}$

ratio test $a_k \geq 0$ ✓

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{(k+1)^2}{3^{(k+1)^2}} \cdot \frac{3^{k^2}}{k^2}$$

$$= \lim_{k\to\infty} \underbrace{\left(\frac{k+1}{k}\right)^2}_{\to 1} \cdot \frac{3^{k^2}}{3^{k^2+2k+1}}$$

$$= \lim_{k\to\infty} \frac{1}{3^{2k+1}} = 0$$

by ratio test the series converges

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9. (15 points) Find the interval of convergence of $\sum_{k=1}^{\infty} \frac{(-1)^k(x-3)^k}{5^k}$. Remember to test the endpoints.

ratio test

$$\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left| \frac{(-1)^{k+1}(x-3)^{k+1}}{5^{k+1}} \cdot \frac{5^k}{(-1)^k(x-3)^k} \right|$$

$$= \lim_{k\to\infty} \frac{1}{5} |x-3|$$

$$= \frac{1}{5}|x-3|$$

converges for $\frac{1}{5}|x-3| < 1$

$$-2 < x < 8$$

endpoints:

$x=-2$ $\sum_{k=1}^{\infty} \frac{(-1)^k(-5)^k}{5^k} = \sum_{k=1}^{\infty} \frac{5^k}{5^k} = \sum_{k=1}^{\infty} 1$ diverges by divergence test $\lim 1 = 1 \neq 0$

$x=8$ $\sum_{k=1}^{\infty} \frac{(-1)^k \cancel{5^k}}{\cancel{5^k}} = \sum_{k=1}^{\infty} (-1)^k$ also diverges by divergence test $\lim (-1)^k \neq 0$

$\Rightarrow$ Interval $= (-2, 8)$

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