

Final**Duration:** 2 hours**Total:** 100 points (to be rescaled to a max of 30% of your total score for the course)

The following rules apply:

- You are expected to abide by the University's rules concerning Academic Honesty.
- You may *not* use your books, notes, or any electronic device including cell phones.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You must complete your work in the space provided.

Check next to your instructor:

Kobotis	
Slutsky	
Dai	
Xie @ 11am	
Xie @ noon	
Heard	
Steenbergen @ noon	
Steenbergen @ 1pm	
Steenbergen @ 2pm	
Woolf	
Cheskidov	
Shvydkoy	

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

(10 pts) **1.** Determine whether the following pairs of vectors are parallel, orthogonal, or neither:

(a) $\mathbf{u} = \langle 2, 1, -1 \rangle$, $\mathbf{v} = \langle -4, -2, 2 \rangle$

(b) $\mathbf{u} = \langle 3, -5 \rangle$, $\mathbf{v} = \langle 1, 3/5 \rangle$

(c) $\mathbf{u} = \langle 2, 0, 0 \rangle$, $\mathbf{v} = \langle 1, 0, 1 \rangle$.

- (10 pts) **2.** An object is thrown horizontally 2 meters above the ground with a speed of 3 m/s. Assuming the gravitational acceleration g is equal to $9m/s^2$, find the total time of flight of the object, and the horizontal distance traveled (range).

(10 pts) **3.** A surface S is given by equation

$$z^2 + xy = 1.$$

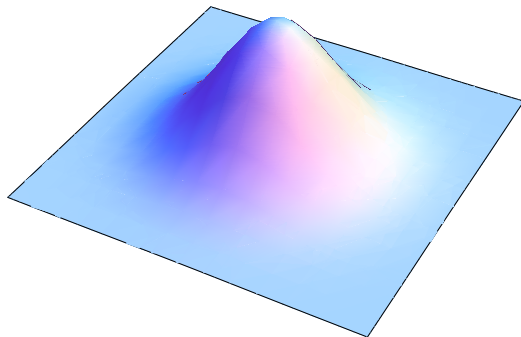
- (a) Verify that S contains point $P(1, 0, 1)$.
- (b) Find an equation of the tangent plane to S at point P .
- (c) Find an equation of the line passing through the point P orthogonal to the surface.

(10 pts) **4.** Find and classify all critical points of the function $f(x, y) = x^4 + 2y^2 - 4xy$.

- (10 pts) **5.** The double integral in this problem can only be evaluated by reversing the order of integration. Sketch the underlying region of integration, reverse the order, and evaluate the obtained integral:

$$\int_0^1 \int_{\sqrt{x}}^1 \frac{10x}{y^5 + 3} dy dx.$$

- (10 pts) **6.** Find the volume of the island bounded above by the graph of $z = e^{-x^2-y^2} - e^{-4}$ and below by the plane $z = 0$.



- (10 pts) **7.** The rectangle C with vertices at $(-1, 0)$, $(-1, 1)$, $(1, 1)$, $(1, 0)$ is oriented counterclockwise. Compute the circulation of the vector field

$$F = \langle \cos(x^2) - xy, \sin(y^2) + y^2 \rangle$$

around the rectangle.

(10 pts) **8.** Compute the surface integral of function $h(x, y, z) = z^2$ over the northern hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$.

(10 pts) **9.** Find a potential of the force field

$$F = \langle yz, xz, xy + z \rangle.$$

Using the found potential compute the work done by the force F when moving an object from point $(1, 1, 2)$ to $(0, 0, 0)$.

- (10 pts) **10.** The roof of a railroad station is given by the graph of the function $z = 1 - y^2$, above the xy -plane and $0 \leq x \leq 3$. It gets bombarded by a storm of hail with velocity field $F = \langle 0, \frac{1}{10}, -1 \rangle$. Compute the downward flux of the hail through the roof.