

Final Exam**Duration:** 2 hours**Total:** 100 points

The following rules apply:

- You are expected to abide by the University's rules concerning Academic Honesty.
- You may *not* use your books, notes, or any electronic device including cell phones.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You must complete your work in the space provided.

Check next to your instructor:

Kobotis	
Lukina @ 11am	
Lukina @ 2pm	
Cole	
Levine	
Steenbergen @ noon	
Steenbergen @ 2pm	
Xie	
Shvydkoy	

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

(10 pts) **1.** The square C with vertices at $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$ is oriented counterclockwise. Compute the circulation of the vector field

$$F = \langle -1 + e^{x^2}, x^2 y^5 + \cos(y^2) \rangle$$

around the square.

(10 pts) **2.** Given two vectors in space

$$\mathbf{u} = \langle 1, -1, 0 \rangle, \quad \mathbf{v} = \langle 1, 0, 1 \rangle$$

find the angle between them; compute the area of the parallelogram formed by the vectors.

- (10 pts) **3.** Find the volume of the region bounded by the cylinder $y = \sin x$ and restricted by $-1 \leq z \leq 1$, $0 \leq x \leq \pi$, $y \geq 0$.

(10 pts) **4.** A surface S is given by equation

$$x = y^2 + z^2 + 1$$

- (a) Describe the surface.
- (b) Verify that the point $P(3, 1, 1)$ belongs to the surface, and find an equation of the tangent plane to S at point P .
- (c) Find all points on the surface where the tangent plane is parallel to the yz -plane.

- (10 pts) **5.** A rectangular window given by $0 \leq y \leq 1$, $0 \leq z \leq 2$, $x = 0$, stands in the wind blowing with velocity field $F = \langle -1, 1, 0 \rangle$. Calculate the flux of the wind through the window relative to the incoming direction (negative direction of the x -axis).

- (10 pts) **6.** Let S be the part of the paraboloid $z = 4 - x^2 - y^2$ above the plane $z = 0$. Find the surface area of S .

- (10 pts) **7.** Find the absolute maximum and minimum of the function $f(x, y) = 1 - 2x^2 - y^2$ over the unit disk $x^2 + y^2 \leq 1$.

(10 pts) **8.** Compute the following double integral

$$\iint_R y dA$$

where R is the region confined between two parabolas $x = y^2$, $x = 2y^2$, and the lines $y = 0$, $y = 1$.

(10 pts) **9.** A force field is given by

$$F = \langle y \cos(xy), x \cos(xy) - 2y \rangle.$$

Examine whether this field is conservative or not. If it is conservative find a potential function. Compute the work done by the force in moving an object from point $(0, 0)$ to point $(\pi/2, 1)$ along a straight line.

(10 pts) **10.** A function is given by

$$h(x, y) = (1 + y) \arctan(x).$$

- (a) Compute the rate of change in the direction of vector $\mathbf{u} = \langle -1, 1 \rangle$ at the origin.
- (b) Determine the direction of the maximum rate of increase at the origin.
- (c) Determine the maximal rate of increase itself at the origin.