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MATH 210 Final Exam

May 5, 2016

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

Name: ______________________________

UIN: ______________________________

University Email: ______________________________

Check next to your instructor's name:

Lukina	12pm	
Lukina	2pm	
Kobotis	8am	
Whyte	10am	
Lenz	11am	
Abernethy	12pm	
Heard	9am	
Wang	2pm	
Kauffman	3pm	

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

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1. **(15 pt)** Given the function $z = f(x, y) = x^2 + xy - e^y$,

   (a) Compute the directional derivative in the direction of the vector $\mathbf{u} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$ at $(1, 0)$.

   (b) Find the direction and the rate of the steepest ascent at $(1, 0)$.

2. **(10 pt)** Let T be a triangle with vertices $A(6,1,2)$, $B(3,6,3)$ and $C(7,0,5)$.

   (a) Find the area of the triangle.

   (b) Find an equation of the plane, containing the triangle.

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3. **(10 pt)** Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y) = 2x + y^2$ subject to the constraint $x^2 + 7y^2 = 64$.

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4. **(10pt)** Find the average value of the function $f(x, y) = xy$ on the rectangle $\{(x, y) \mid 2 \leq x \leq 4, 0 \leq y \leq 1\}$.

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5. **(15 pt)** Evaluate the integral

$$\iiint_D x^2 + y^2 \, dV,$$

where D is the solid bounded by the cylinder with the circle base given by the equation $x^2 + y^2 = 1$ and the planes $z = 0$ and $x + y + z = 5$.

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6. **(10 pt)** Compute the curl and the divergence of the vector field

$$\mathbf{F}(x, y, z) = \langle xy + z, x - y, z - x^2 y \rangle.$$

7. **(10 pt)** The vector field $\mathbf{F}(x, y, z) = \langle 2xy + yz, x^2 + xz, xy \rangle$ is conservative.

(a) Find a potential function for $\mathbf{F}$.

(b) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is a curve originating at $(1, 1, 0)$ and terminating at $(2, -1, 1)$.

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8. **(10pt)** Find the circulation of the vector field $\mathbf{F}(x, y) = \langle 2x + 3y, yx + e^{\cos y} \rangle$ on the triangular curve with vertices $(0, 0)$, $(0, 1)$ and $(3, 0)$ oriented counterclockwise. (Hint: use Green's theorem.)

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9. **(10pt)** Compute the flux of the vector field $\mathbf{F} = \langle 0, -z, 3y \rangle$ across the surface in the first octant given by the equation $x + y + z = 2$, in the direction of the positive z-semiaxis.