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MATH 210 Final Exam

May 10, 2018

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

Check next to your instructor's name:

Lukina			Braithwaite	
Cameron			Kobotis	
Abramov			Shulman	
Heard			Woolf	
Skalit			Freitag	

1. **(10 pt)** Find the equation of the tangent plane to the graph

$$z = x^3 - 2\sin(y)$$

at the point $(1, 0, 1)$.

2. **(10 pt)** The velocity of a particle moving in space is given by

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle \sqrt{t}, \cos(\pi t), 4t \rangle.$$

At the point of time $t = 1$ the particle has coordinates $\mathbf{r}(1) = \langle 2, 3, 4 \rangle$. Find the position function $\mathbf{r}(t)$ of the particle.

3. **(10 pt)** Consider the function

$$f(x, y) = x^3 - y^2 - xy + 1.$$

Find all the critical points of f and classify them as local maxima, local minima or saddle points.

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4. **(10pt)** Use the method of Lagrange multipliers to find the maximum value of the function

$$f(x, y, z) = 2x - 3y - 4z$$

subject to the constraint

$$2x^2 + y^2 + z^2 = 16.$$

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5. **(10 pt)** Consider the solid D in the first octant enclosed by the cylinder $x^2 + y^2 = 4$ and the plane $z = 3$. Compute $\displaystyle\iiint_D x\,dV$.

6. **(10 pt)** Consider a conservative vector field $\mathbf{F} = \langle 3x^2z, z^2, x^3 + 2yz \rangle$.

   (a) Find a potential function for $\mathbf{F}$.

   (b) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is a path from the point $A(1,1,0)$ to $B(2,1,1)$.

7. **(10pt)** Let R be the region in the first quadrant of the plane bounded by the lines $y = 0$, $x = 0$, $y = 2$ and $x = 3$. Let C be the boundary of R, equipped with counter-clockwise orientation. Consider the vector field $\mathbf{F} = \langle xy^2, x \rangle$.

   (a) Sketch C and show its orientation with an arrow.

   (b) Use Green's Theorem to compute the circulation $\oint_C \mathbf{F} \cdot d\,\mathbf{r}$.

   (c) Use Green's Theorem to compute the flux $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds$.

8. **(10 pt)** Consider the vector field $\mathbf{F}(x, y, z) = \langle xz, e^z - yz, \cos x \rangle$.

   (a) Find the curl of $\mathbf{F}$.

   (b) Find the divergence of $\mathbf{F}$.

   (c) Determine if the vector field $\mathbf{F}$ is conservative. Justify your answer.

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9. **(10 pt)** Compute the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$, where S is the surface

$$z = \frac{1}{4}x^2 + \frac{1}{4}y^2,$$

over the rectangle $\{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 2\}$, oriented with the downward normal, and $\mathbf{F} = \langle 1, 3, 5 \rangle$.

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10. **(10 pt)** Let S denote the plane $2x + y + 3z = 6$ in the first octant with the upward normal, and C denote its triangular boundary. Use Stokes' Theorem to evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F} = \langle 2z - x, x + y + z, 2y - x \rangle.$$