MATH 210
Sample exam problems for the final exam
Fall 2009

This is a list of problems to help you prepare for the final exam. It is not considered to be exhaustive and you should not expect to find your actual exam problems in the list below. It is to serve as a study aid and it cannot be a substitute for in-class reviews or study of your class notes.

1. A triangle has vertices at the points

$$A = (1, 1, 1), \quad B = (1, -3, 4), \quad \text{and} \quad C = (2, -1, 3)$$

   (a) Find the cosine of the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

   (b) Find an equation of the plane containing the triangle.

2. Find the critical points of the function:

$$f(x, y) = x^2 + y^2 + x^2 y + 1$$

   and classify each point as corresponding to either a saddle point, a local minimum, or a local maximum.

3. Find the directional derivative of the function $f(x, y) = e^x \sin(xy)$ at the point $(0, \pi)$ in the direction of $\vec{\mathbf{v}} = \langle 1, 0 \rangle$. In the direction of which unit vector is f increasing most rapidly at the point $(0, \pi)$?

4. Consider a space curve whose parametrization is given by:

$$\vec{\mathbf{r}}(t) = \left\langle \cos(\pi t), t^2, 1 \right\rangle$$

   Find the unit tangent vector and curvature when $t = 2$.

5. Evaluate $\displaystyle\iint_{\mathcal{D}} e^{-(x^2+y^2)}\, dA$ where $\mathcal{D} = \{(x, y) : \ x^2 + y^2 \leq 1, \ x \geq 0, \ y \geq 0\}$.

6. Evaluate $\displaystyle\int_{\mathcal{C}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$ where $\vec{\mathbf{F}} = \langle y + z, z + x, x + y \rangle$ and $\mathcal{C}$ is the line segment from $(1, 1, 0)$ to $(2, 0, -1)$.

7. Consider the paraboloid $z = 4 - x^2 - y^2$.

   (a) Find an equation for the tangent plane to the paraboloid at the point $(1, 2, -1)$.

   (b) Find the volume that is bounded by the paraboloid and the plane $z = 0$.

8. Let B be a constant and consider the vector field defined by:

$$\overrightarrow{\mathbf{F}} = \big\langle B\,xy + 1, x^2 + 2y \big\rangle$$

   (a) For what value of B can we write $\overrightarrow{\mathbf{F}} = \overrightarrow{\nabla}\varphi$ for some scalar function φ? Find such a function φ in this case.

   (b) Using the value of B you found in part (a), evaluate the line integral of $\overrightarrow{\mathbf{F}}$ along any curve from $(1, 0)$ to $(-1, 0)$.

9. Consider $f(x, y) = x\sin(x + 2y)$.

   (a) Compute the partial derivatives f_x, f_y, f_{xx}, f_{xy}, and f_{yy}.

   (b) If $x = s^2 + t$ and $y = 2s + t^3$, compute the partials f_s and f_t.

10. Find the points on the ellipse $x^2 + xy + y^2 = 9$ where the distance from the origin is maximal and minimal. (Hint: Let $f(x, y) = x^2 + y^2$ be the function you want to extremize where (x, y) is a point on the ellipse.)

11. Sketch the region of integration for the integral below and evaluate the integral.

$$\int_0^1 \int_y^1 e^{-x^2}\,dx\,dy$$

12. Evaluate $\displaystyle\int_{\mathcal{C}} f(x, y, z)\,ds$ where $f(x, y, z) = z\sqrt{x^2 + y^2}$ and $\mathcal{C}$ is the helix $\overrightarrow{\mathbf{c}}(t) = (4\cos t, 4\sin t, 3t)$ for $0 \le t \le 2\pi$.

13. Consider the vectors $\overrightarrow{\mathbf{v}} = \langle 1, 2, a\rangle$ and $\overrightarrow{\mathbf{w}} = \langle 1, 1, 1\rangle$.

   (a) Find the value of a such that $\overrightarrow{\mathbf{v}}$ is perpendicular to $\overrightarrow{\mathbf{w}}$.

   (b) Find the two values of a such that the area of the parallelogram determined by $\overrightarrow{\mathbf{v}}$ and $\overrightarrow{\mathbf{w}}$ is equal to $\sqrt{6}$.

14. Consider a particle whose position vector is given by:

$$\overrightarrow{\mathbf{r}}(t) = \big\langle \sin(\pi t), t^2, t + 1 \big\rangle$$

(a) Find the velocity $\overrightarrow{\mathbf{r}}'(t)$ and acceleration $\overrightarrow{\mathbf{r}}''(t)$.

(b) Set up the integral you would compute to find the distance traveled by the particle from $t = 0$ to $t = 4$. **Do not attempt to compute the integral**.

15. Find the volume of the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 4$.

16. Use Green's Theorem to compute $\oint_{\mathcal{C}} xy\,dx + y^5\,dy$ where $\mathcal{C}$ is the boundary of the triangle with vertices at $(0,0)$, $(2,0)$, and $(2,1)$, oriented counterclockwise.

17. Consider the plane P containing the points $A = (1,0,0)$, $B = (2,1,1)$, and $C = (1,0,2)$.

    (a) Find a unit vector perpendicular to P.

    (b) Find the intersection of P with the line perpendicular to P that contains the point $D = (1,1,1)$.

18. Use a triple integral to compute the volume of the region below the sphere $x^2+y^2+z^2 = 4$ and above the disk $x^2 + y^2 \leq 1$ in the xy-plane.

19. Consider the cone $z = \sqrt{x^2 + y^2}$ for $0 \leq z \leq 4$.

    (a) Write a parametrization $\Phi(u, v)$ for the cone, clearly indicating the domain of Φ.

    (b) Find the surface area of the cone.

20. Calculate $\int_{\mathcal{C}} y\,dx + (x + z)\,dy + y\,dz$ along the curve $\mathcal{C}$ given by $\overrightarrow{\mathbf{c}}(t) = (t, t^2, t^3)$ for $0 \leq t \leq 1$.