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MATH 210 Exam 1

October 5, 2017

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.

- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

Check next to your instructor's name:

Lukina			Steenbergen	
Hong			Kobotis	
Dai			Bona	
Sinapova			Kashcheyeva	
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Nuer				

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1. **(15pt)**

   (a) Find an equation for a vector-valued function $\mathbf{r}(t)$ describing the line through the points $P(1,3,2)$ and $Q(-3,1,4)$.

   (b) Find the coordinates of the point of the intersection of the line in a) and the plane $z=4$.

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2. **(15pt)** Consider the line $\mathbf{r}(t) = \langle -2 - 5t, 3 + 2t, 4t \rangle$ and the point $P(2, -2, -1)$. Find an equation of the plane which contains the line $\mathbf{r}(t)$ and the point P.

3. **(15 pt)** A particle moves in space with acceleration given by the vector-valued function

$$\mathbf{a}(t) = \langle 9\sin 3t, 6t^2, 9\cos 3t \rangle.$$

Time t is measured in seconds. At time $t = 0$ the particle is stationary, that is, $\mathbf{v} = \langle 0, 0, 0 \rangle$, and located at the point with coordinates $(2, -5, 3)$.

(a) Compute the velocity function $\mathbf{v}(t)$.

(b) Compute the position function $\mathbf{r}(t)$.

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4. **(10 pt)** Let $z = f(x, y) = \sqrt{x^2 - 1 - y}$.

   (a) State the domain of f. Give a rough sketch of this region in the xy plane; be sure to shade in the area belonging to the domain.

   (b) Find the equations of level curves for $z = 1$ and for $z = \sqrt{2}$. Sketch the level curves on the same picture as the domain of f.

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5. **(10 pt)** Let $\mathbf{r}(t) = \langle t^2 - 1,\, 3t^2 - 6,\, t^2 + 2t \rangle$.

   (a) Compute $\mathbf{r}'(t)$.

   (b) Compute the tangent vector for $t = 1$.

   (c) Compute $\int_0^1 \mathbf{r}(t)\, dt$.

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6. **(10 pt)** A surface is given by the equation

$$z = \sqrt{x^2 + y^2}.$$

(a) Find the equations of the xy-, yz- and xz-traces of the surface.

(b) Find the equation of the trace in the plane $z = 1$.

(c) Sketch the surface using the traces you found in a) and b).

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7. **(10 pt)** Find all four second order partial derivatives of the function

$$f(x, y) = x^2 - xy^2$$

and evaluate them at the point $(2, -1)$.

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8. **(15 pt)** Consider the function $f(x, y) = x + xy + y^2$.

   (a) Compute the gradient $\nabla f(x, y)$.

   (b) Find the directional derivative $D_{\mathbf{u}} f(1, 1)$, where $\mathbf{u} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$.

   (c) Compute the unit vector in the direction of maximum increase at $(1, 1)$.

   (d) What is the rate of maximum increase in c)?