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MATH 210 Exam 1

February 22, 2018

Directions. Fill in each of the lines below. Then read the directions that follow before beginning the exam. YOU MAY NOT OPEN THE EXAM UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

First name (please write as legibly as possible within the boxes)

Last name

NetId (the part of the email before @uic.edu)

- All of your work must fit within the boxes on each page for each question. Nothing outside of the box will be graded! If you write outside of the box, there is a good chance that your solution will not be read and therefore not graded.
- A solution for one problem may not go on another page.
- Show all your work. Unjustified answers are not correct. Make clear what your final answer is.
- Have your student ID ready to be checked when submitting your exam.

Check next to your instructor's name:

Lukina			Braithwaite	
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Abramov			Shulman	
Heard			Woolf	
Skalit			Freitag	

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1. **(15pt)** Consider the points $P(1,1,3)$, $Q(4,0,1)$, and $R(0,1,6)$.

   (a) Find the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

   (b) Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

   (c) Find the area of the triangle with vertices P, Q, and R.

a) $\overrightarrow{PQ} = \langle 4-1, 0-1, 1-3 \rangle = \langle 3, -1, -2 \rangle$

$\overrightarrow{PR} = \langle 0-1, 1-1, 6-3 \rangle = \langle -1, 0, 3 \rangle$

b) $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & -2 \\ -1 & 0 & 3 \end{vmatrix} = \vec{i}(-3-0) - \vec{j}(9-2) + \vec{k}(0-1) =$

$\langle -3, -7, -1 \rangle$

c) area of $\triangle PQR = \frac{1}{2} \left| \overrightarrow{PQ} \times \overrightarrow{PR} \right| =$

$\frac{1}{2}\sqrt{9+49+1} = \frac{1}{2}\sqrt{59}$

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2. **(15pt)** Consider the vectors $\mathbf{u} = \langle 1, 1, 0 \rangle$ and $\mathbf{v} = \langle 1, -2, 0 \rangle$.

   (a) Compute $3\mathbf{u} - 2\mathbf{v}$.

   (b) Find the dot product $\mathbf{u} \cdot \mathbf{v}$.

   (c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$. Leave your answer in exact form.

a) $3\vec{u} - 2\vec{v} = 3\langle 1,1,0\rangle - 2\langle 1,-2,0\rangle =$

$\langle 3,3,0\rangle - \langle 2,-4,0\rangle = \langle 1,7,0\rangle$

b) $\vec{u} \cdot \vec{v} = \langle 1,1,0\rangle \cdot \langle 1,-2,0\rangle =$

$1 - 2 + 0 = -1$

c) $|\vec{u}| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$ $\quad |\vec{v}| = \sqrt{1+4+0} = \sqrt{5}$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{-1}{\sqrt{5}\sqrt{2}} = -\frac{1}{\sqrt{10}}$$

$$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{10}}\right)$$

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3. **(15 pt)** Suppose the position of a particle in the plane is given by $\mathbf{r}(t)$. It is known that its acceleration is $\mathbf{r}''(t) = \langle 9\cos(3t), 6t - 2\rangle$.

   (a) Compute the velocity function $\mathbf{v}(t)$ subject to the condition that $\mathbf{v}(0) = \langle 0, 1\rangle$.

   (b) Compute the position function $\mathbf{r}(t)$ subject to the condition that $\mathbf{r}(0) = \langle 2, 0\rangle$.

a) $\vec{v}(t) = \vec{r}'(t) = \int \vec{r}''(t)\,dt = \left\langle 9 \cdot \frac{1}{3}\sin(3t),\ 6\frac{t^2}{2} - 2t \right\rangle +$

$\langle C_1, C_2 \rangle = \langle 3\sin(3t),\ 3t^2 - 2t \rangle +$

$\langle C_1, C_2 \rangle$

$\vec{v}(0) = \langle C_1, C_2 \rangle = \langle 0, 1 \rangle,$

$C_1 = 0, \quad C_2 = 1$

$\vec{v}(t) = \langle 3\sin(3t),\ 3t^2 - 2t + 1 \rangle$

b) $\vec{r}(t) = \int \vec{v}(t)\,dt = \left\langle 3 \cdot \frac{1}{3}(-\cos(3t)) + D_1,\right.$

$\left. 3\frac{t^3}{3} - 2\frac{t^2}{2} + t + D_2 \right\rangle =$

$\langle -\cos(3t) + D_1,\ t^3 - t^2 + t + D_2 \rangle$

$\vec{r}(0) = \langle -1 + D_1, D_2 \rangle = \langle 2, 0 \rangle$

$-1 + D_1 = 2, \quad D_1 = 3$

$D_2 = 0$

$\vec{r}(t) = \langle -\cos(3t) + 3,\ t^3 - t^2 + t \rangle$

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4. **(10 pt)** Let $h(x,y) = \sqrt{3x - y + 4}$.

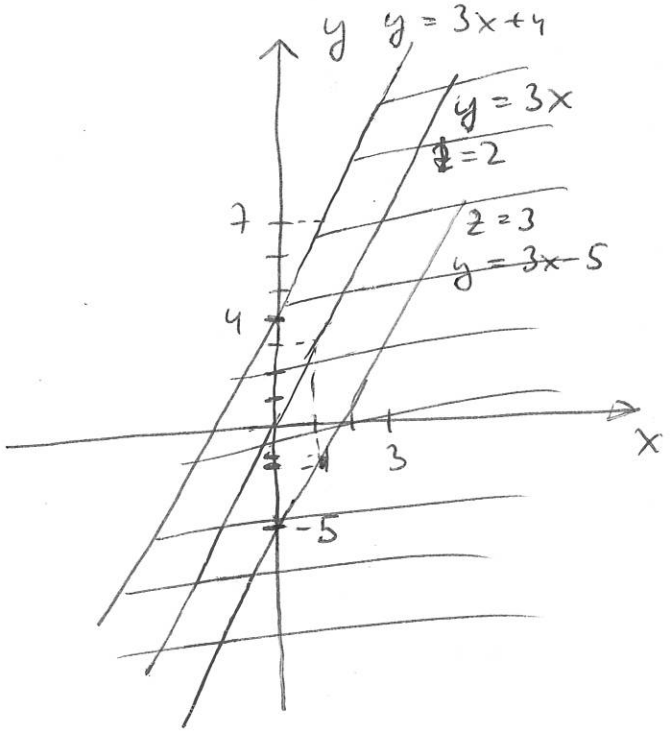
   (a) State the domain of h. Give a rough sketch of this region in the xy plane; be sure to shade in the area belonging to the domain.

   (b) Find the equations of level curves for $z = 2$ and for $z = 3$. Sketch the level curves on the same picture as the domain of f. Label the level curves by the corresponding value of z.

a) $D = \{(x,y) \mid 3x - y + 4 \geq 0\}$

$3x - y + 4 = 0$

$y = 3x + 4$



b) $z = 2$:

$2 = \sqrt{3x - y + 4}$

$4 = 3x - y + 4$

$y = 3x$

$z = 3$:

$3 = \sqrt{3x - y + 4}$

$9 = 3x - y + 4$

$y = 3x - 5$

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5. **(10 pt)** Let

$$\mathbf{r}(t) = \langle 4\cos(t), e^{2t}, t^5 - t \rangle.$$

(a) Compute $\mathbf{r}'(t)$ and $|\mathbf{r}'(t)|$.

(b) Find the unit tangent vector to $\mathbf{r}(t)$ at $t = 0$.

(c) Compute $\int_0^1 \mathbf{r}(t)\,dt$.

a) $\vec{r}'(t) = \langle -4\sin t, 2e^{2t}, 5t^4 - 1 \rangle$

$$|\vec{r}'(t)| = \sqrt{16\sin^2 t + 4e^{4t} + (5t^4-1)^2}$$

b) $\vec{r}'(0) = \langle 0, 2, -1 \rangle$

$$|\vec{r}'(0)| = \sqrt{4+1} = \sqrt{5}$$

$$\vec{u} = \frac{\vec{r}'(0)}{|\vec{r}'(0)|} = \left\langle 0, \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$$

c) $\int_0^1 \vec{r}(t)\,dt = \left\langle 4\sin t, \frac{1}{2}e^{2t}, \frac{t^6}{6} - \frac{t^2}{2} \right\rangle \Big|_0^1 =$

$$\left\langle 4\sin 1, \frac{1}{2}e^2 - \frac{1}{2}, \frac{1}{6} - \frac{1}{2} \right\rangle$$

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6. **(10 pt)** Give a vector equation of the line in the intersection of the planes $x + y + z = 3$ and $x + y - z = 0$.

$\vec{n}_1 = \langle 1, 1, 1 \rangle$

$\vec{n}_2 = \langle 1, 1, -1 \rangle$

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \vec{i}(-1-1) - \vec{j}(-1-1) + \vec{k}(1-1) =$$

$$\langle -2, 2, 0 \rangle$$

$x+y+z = 3$

$x+y-z = 0$

$2(x+y) = 3$

$x+y = \frac{3}{2}$,

take $y = 0$, then $x = \frac{3}{2}$,

$z = x+y = \frac{3}{2} + 0 = \frac{3}{2}$,

$\left(\frac{3}{2}, 0, \frac{3}{2}\right)$

$$\vec{r}(t) = \left\langle \frac{3}{2}, 0, \frac{3}{2} \right\rangle + t\langle -2, 2, 0 \rangle$$

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7. **(10 pt)** Find all first and second order partial derivatives of the function

$$f(x, y) = \sin(x^2 y).$$

Evaluate the first and the second order partial derivatives at the point $(0, \pi)$.

$f_x = \cos(x^2y) \cdot 2xy \qquad f_y = \cos(x^2y)\, x^2$

$f_{xx} = 2y \cos(x^2y) + 2xy(-\sin(x^2y))\, 2xy$

$f_{xy} = 2x \cos(x^2y) + 2xy(-\sin(x^2y))\, x^2$

$f_{yx} = 2x \cos(x^2y) + x^2(-\sin(x^2y))\, 2xy$

$f_{yy} = -x^2 \sin(x^2y)\, x^2$

$f_x(0, \pi) = \cos 0 \cdot 2 \cdot 0 \cdot \pi = 0$

$f_y(0, \pi) = 0$

$f_{xx}(0, \pi) = 2 \cdot \pi \cos 0 + 2 \cdot 0 \cdot \pi(-\sin 0) \cdot 2 \cdot 0 \cdot \pi = 2\pi$

$f_{xy}(0, \pi) = f_{yx}(0, \pi) = 0$

$f_{yy}(0, \pi) = 0$

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8. **(15 pt)** Consider the function $f(x,y) = 3xy - y^2 + x$.

(a) What are the partial derivatives of f at $(1,-1)$?

(b) What is the unit vector in the direction of greatest increase of f at $(1,-1)$?

(c) What is the directional derivative of f at $(1,-1)$ in this direction?

a) $\nabla f(x,y) = \langle 3y+1, 3x-2y \rangle$

$\nabla f(1,-1) = \langle -3+1, 3\cdot 1 - 2\cdot(-1) \rangle = \langle -2, 5 \rangle$

b) $|\nabla f(1,-1)| = \sqrt{4+25} = \sqrt{29}$

$\vec{u} = \dfrac{\nabla f(1,-1)}{|\nabla f(1,-1)|} = \left\langle -\dfrac{2}{\sqrt{29}}, \dfrac{5}{\sqrt{29}} \right\rangle$ -

unit vector in the direction of greatest increase

c) $D_{\vec{u}} f(1,-1) = |\nabla f(1,-1)| = \sqrt{29}$