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1. **(20 pt)** Let $f(x, y) = \ln(2x + y)$.

   (a) Write the equation of the tangent plane to $f(x, y)$ at $(-1, 3)$.

   (b) Use part (a) to estimate $f(-1.1, 2.9)$.

a) $f_x = \dfrac{2}{2x+y}$ $\quad f_y = \dfrac{1}{2x+y}$

$f_x(-1,3) = \dfrac{2}{-2+3} = 2$

$f_y(-1,3) = \dfrac{1}{-2+3} = 1$

$f(-1,3) = \ln(2(-1)+3) = \ln 1 = 0$

$z = 2(x+1) + 1(y-3) + 0 = 2(x+1) + 1(y-3).$

b) $L(x,y) = 2(x+1) + (y-3)$

$L(-1.1, 2.9) = 2(-1.1+1) + (2.9-3) =$

$-0.2 - 0.1 = -0.3$

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2. **(15pt)** For the function

$$f(x, y) = x^3 - 12x + y^2 - 4y + 1,$$

find the critical points and classify them as local minima, local maxima, or saddle points.

$f_x = 3x^2 - 12 = 0$ $\quad x^2 = 4, \quad x = \pm 2$

$\Rightarrow$

$f_y = 2y - 4 = 0$ $\quad y = 2$

points $(-2, 2), \quad (2, 2)$

$f_{xx} = 6x$

$f_{xy} = 0$

$f_{yy} = 2$

$D(x, y) = 6x \cdot 2 - 0^2 = 12x$

$D(-2, 2) = 12(-2) < 0$ saddle point

$D(2, 2) = 12 \cdot 2 > 0$

$f_{xx}(2, 2) = 6 \cdot 2 > 0$ local min

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3. **(15pt)** Use the method of Lagrange multipliers to find the minimum and the maximum of the function

$$f(x, y) = x - 2y$$

on the circle $x^2 + y^2 = 1$.

$\nabla f(x,y) = \langle 1, -2 \rangle$

$g(x,y) = x^2 + y^2 - 1 = 0$

$\nabla g(x,y) = \langle 2x, 2y \rangle$

$1 = 2\lambda x$

$-2 = 2\lambda y \quad \Rightarrow \quad x = \frac{1}{2\lambda}$

$x^2 + y^2 = 1 \qquad y = -\frac{2}{2\lambda}$

$\frac{1}{4\lambda^2} + \frac{4}{4\lambda^2} = 1$

$\frac{5}{4\lambda^2} = 1 \quad \Rightarrow \quad \lambda = \pm \frac{\sqrt{5}}{2}$

$\lambda = -\frac{\sqrt{5}}{2}$: $x = -\frac{1}{\sqrt{5}}$, $y = \frac{2}{\sqrt{5}}$,

$f\left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = -\frac{1}{\sqrt{5}} - \frac{4}{\sqrt{5}} = -\frac{5}{\sqrt{5}} = -\sqrt{5}$

$\lambda = \frac{\sqrt{5}}{2}$: $x = \frac{1}{\sqrt{5}}$, $y = -\frac{2}{\sqrt{5}}$

$f\left(\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right) = \frac{1}{\sqrt{5}} + \frac{4}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$

max $f\left(\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right) = \sqrt{5}$

min $f\left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = -\sqrt{5}$

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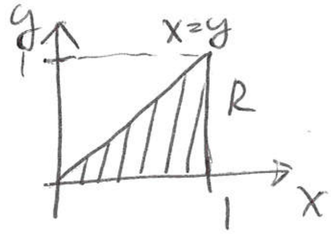
4. **(15 pt)** For the integral

$$\int_0^1 \int_y^1 \cos(x^2)\, dx\, dy$$

- Sketch the region of integration.
- Change the order of integration.
- Evaluate the integral.

a) $R = \{(x,y) \mid 0 \le y \le 1,\ y \le x \le 1\}$



b) $R = \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le x\}$

$$\int_0^1 \int_y^1 \cos(x^2)\, dx\, dy = \int_0^1 \int_0^x \cos x^2\, dy\, dx =$$

$$\int_0^1 y \cos x^2 \Big|_0^x dx = \int_0^1 x \cos x^2\, dx =$$

$u = x^2 \quad du = 2x\,dx$

if $x = 0$ then $u = 0$

$x = 1 \quad u = 1$

$$\frac{1}{2}\int_0^1 \cos u\, du = \frac{1}{2} \sin u \Big|_0^1 = \frac{1}{2} \sin 1 -$$

$$\frac{1}{2} \sin 0 = \frac{1}{2} \sin 1.$$

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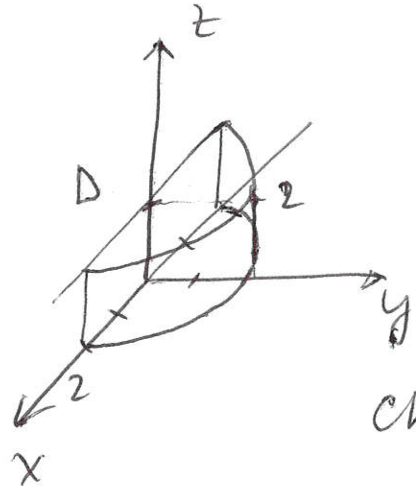
5. **(15 pt)** Evaluate the iterated integral by converting to cylindrical coordinates

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{1}(x^2+y^2)\,dz\,dy\,dx$$



$D = \{(x,y,z) \mid -2 \le x \le 2,\ 0 \le y \le \sqrt{4-x^2},\ 0 \le z \le 1\}$

$\left.\begin{array}{l} y = \sqrt{4-x^2} \\ x^2+y^2 = 4 \end{array}\right\}$ upper half of a circle of radius 1

Change to cylindrical coordinates:

$x^2+y^2 = r^2$,

$D = \{(r,\theta,z) \mid 0 \le \theta \le \pi,\ 0 \le r \le 2,\ 0 \le z \le 1\}$

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{1}(x^2+y^2)\,dz\,dy\,dx = \int_{0}^{\pi}\int_{0}^{2}\int_{0}^{1} r^2\, r\,dz\,dr\,d\theta =$$

$$\int_{0}^{\pi}\int_{0}^{2} r^3 z\Big|_0^1\,dr\,d\theta = \int_{0}^{\pi}\int_{0}^{2} r^3\,dr\,d\theta =$$

$$\int_{0}^{\pi}\frac{r^4}{4}\Big|_0^2\,d\theta = \int_{0}^{\pi}\left(\frac{16}{4}-0\right)d\theta =$$

$$4\theta\Big|_0^{\pi} = 4\pi$$

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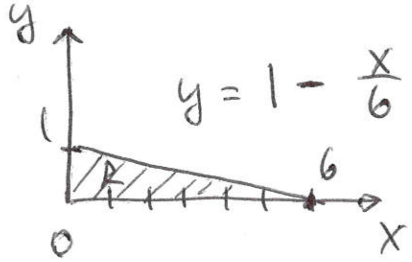
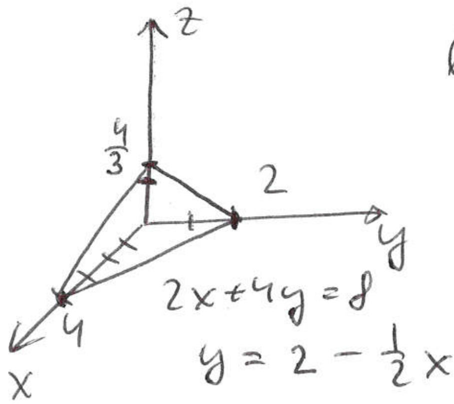
6. **(20 pt)** a) Write down an iterated double integral which expresses the area of the triangular region with vertices $(0,0)$, $(6,0)$ and $(0,1)$. Do not evaluate the integral.



$$R = \{(x,y) \mid 0 \le x \le 6,\ 0 \le y \le 1 - \tfrac{x}{6}\}$$

$$\text{area} = \iint_R dA = \int_0^6 \int_0^{1-\frac{x}{6}} dy\,dx$$

b) Write down an iterated triple integral that expresses the volume of the tetrahedron bounded by the xy-plane, yz-plane, xz-plane and the plane $2x + 4y + 6z = 8$. Do not evaluate the integral.



$$6z = 8 - 2x - 4y$$

$$z = \frac{1}{6}(8 - 2x - 4y)$$

$$D = \{(x,y,z) \mid 0 \le x \le 4,\ 0 \le y \le 2 - \tfrac{1}{2}x,\ 0 \le z \le \tfrac{1}{6}(8-2x-4y)\}$$

$$V = \iiint_D dV = \int_0^4 \int_0^{2-\frac{1}{2}x} \int_0^{\frac{1}{6}(8-2x-4y)} dz\,dy\,dx$$