Math 210 – Spring 2015 Name: ____________________

Midterm Exam 2
Duration: 2 hours
Total: 70 points

The following rules apply:

- You are expected to abide by the University's rules concerning Academic Honesty.
- You may *not* use your books, notes, or any electronic device including cell phones.
- You must show all of your work. An answer, right or wrong, without the proper justification will receive little to no credit.
- You must complete your work in the space provided.

Check next to your instructor:

Kobotis	A001	
Lukina @ 11am	B101	
Lukina @ 2pm	B101	
Cole	A001	
Levine	A001	
Steenbergen @ noon	C001	
Steenbergen @ 2pm	C001	
Xie	B101	
Shvydkoy	B101	

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total	70	

(10 pts) **1.** A function $z(x,y)$ is given implicitly by

$$\sin(x+z) + e^{yz} = 1.$$

Use the method of implicit differentiation to evaluate partial derivatives z_x, z_y at the point $(0,0,0)$.

To compute z_x differentiate the equation in x keeping in mind that $z = z(x,y)$:

$$\cos(x+z)\,(1+z_x) + y z_x e^{yz} = 0$$

$$z_x\left[\cos(x+z) + y e^{yz}\right] = -\cos(x+z)$$

$$z_x = -\frac{\cos(x+z)}{\cos(x+z)+ye^{yz}}$$

Similarly,

$$\cos(x+z)\, z_y + (z + y z_y)\, e^{yz} = 0$$

$$z_y\left[\cos(x+z) + y e^{yz}\right] = -z e^{yz}$$

$$z_y = -\frac{z e^{yz}}{\cos(x+z)+ye^{yz}}.$$

(10 pts) **2.** Find an equation of the plane tangent to the surface

$$F = xyz - \ln z = 0$$

at point $(0, 1, 1)$.

$$F_x = yz \quad = 1$$

$$F_y = xz \quad = 0$$

$$F_z = xy - \frac{1}{z} \quad = -1$$

$(0,1,1)$

So,

$$1 \cdot (x-0) + 0 \cdot (y-1) - 1 \cdot (z-1) = 0$$

$$\boxed{x - z + 1 = 0}$$

(10 pts) **3.** Altitude of a terrain is given by the function

$$A(x,y) = 4 - 2x^2 - y^2 + xy.$$

A hiker standing at the point $(1, 1, 2)$ wants to find the direction of the steepest climb. Find that direction and compute the maximal rate of the climb.

Steepest climb is in the direction of the gradient:

$$\left.\begin{aligned} A_x &= -4x + y \\ A_y &= -2y + x \end{aligned}\right|_{(1,1)} \begin{aligned} &= -3 \\ &= -1 \end{aligned}$$

$$\nabla A = \langle -3, -1 \rangle$$

$$\text{Rate} = |\nabla A| = \sqrt{9+1} = \sqrt{10}$$

(10 pts) **4.** Find and classify all critical points of the function

$$f(x, y) = 2x^2 + y^4 - 4xy$$

on the entire plane. Indicate which method you are using.

$f_x = 4x - 4y = 0 \rightarrow x = y$

$f_y = 4y^3 - 4x = 0 \rightarrow y^3 - y = 0$

$y(y-1)(y+1) = 0$

$y = 0, 1, -1$

$x = 0, 1, -1$

3 points: $(0,0), (1,1), (-1,-1)$.

$f_{xx} = 4$, $f_{xy} = -4$, $f_{yy} = 12y^2$

We use Second Derivative Test

$D = 48y^2 - 16$

	D	f_{xx}	
$(0,0)$	-16	4	saddle
$(1,1)$	32	4	min
$(-1,-1)$	32	4	min

(10 pts) **5.** Any point P outside of the parabolic cylinder $z = y^2$ has a point on the cylinder closest to P. Let $P = (1, 0, 1)$. Find a point on the parabolic cylinder closest to P. And compute the minimal distance.

$f = (x-1)^2 + y^2 + (z-1)^2$ – square distance

$g = y^2 - z = 0$ – constraint

$\nabla f = \lambda \nabla g$

$$\begin{cases} 2(x-1) = 0 & \longrightarrow \quad x = 1 \\ 2y = 2y\lambda & \longrightarrow \quad y = 0 \quad \text{or} \quad \lambda = 1 \\ 2(z-1) = -\lambda & \qquad\quad \Downarrow \qquad\qquad\quad \Downarrow \\ y^2 = z & \qquad\quad z = 0 \qquad\quad z = \frac{1}{2} \end{cases}$$

$(1, 0, 0)$ $\qquad$ $y = \pm \frac{1}{\sqrt{2}}$

We found 3 critical point:

$(1, 0, 0)$; $(1, \frac{1}{\sqrt{2}}, \frac{1}{2})$; $(1, -\frac{1}{\sqrt{2}}, \frac{1}{2})$

$f = 1$ $\qquad$ $f = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

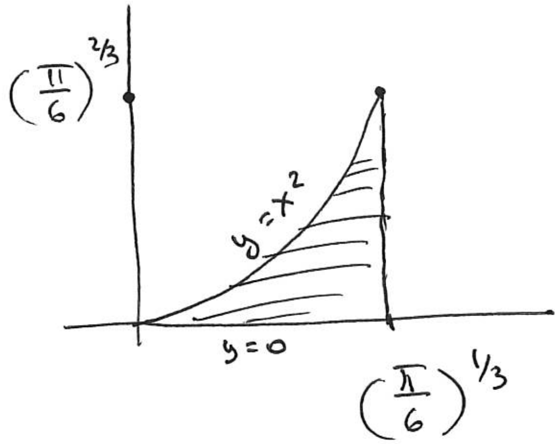
– this is the smallest value

$$\boxed{\begin{array}{l} \text{dist} = \sqrt{3/4} \\ \text{at } (1, \pm\frac{1}{\sqrt{2}}, \frac{1}{2}) \end{array}}$$

(10 pts) **6.** Change the order of integration in the double integral

$$\int_0^{\left(\frac{\pi}{6}\right)^{2/3}} \int_{\sqrt{y}}^{\left(\frac{\pi}{6}\right)^{1/3}} \cos(x^3)\, dx\, dy.$$

Compute the resulting integral.



$x=\sqrt{y}$

$y=x^2$

$$\iint = \int_0^{\left(\frac{\pi}{6}\right)^{1/3}} \int_0^{x^2} \cos(x^3)\, dy\, dx$$

$$= \int_0^{\left(\pi/6\right)^{1/3}} x^2 \cos(x^3)\, dx$$

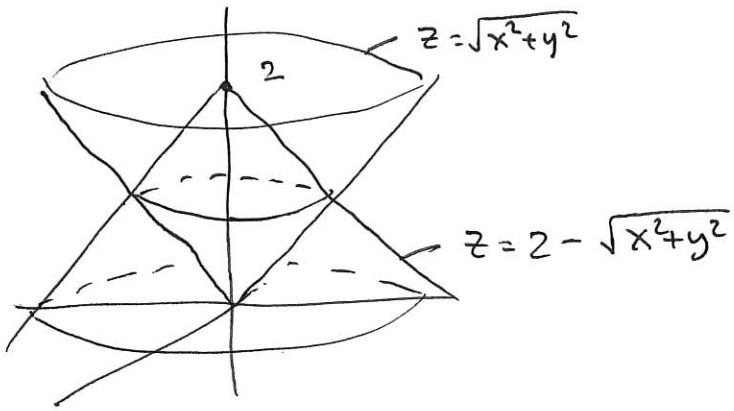
$$= \left(u = x^3, \quad du = 3x^2 dx \right)$$

$$= \frac{1}{3} \int_0^{\pi/6} \cos(u)\, du$$

$$= \frac{1}{3} \sin(u) \Big|_0^{\pi/6} = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

(10 pts) **7.** Compute the volume of the region enclosed between two cones

$$z = \sqrt{x^2 + y^2} \text{ and } z = 2 - \sqrt{x^2 + y^2}.$$



Intersection: $2 - \sqrt{x^2+y^2} = \sqrt{x^2+y^2}$

$$x^2+y^2 = 1$$

In polar coordinates

$$\text{Vol} = \int_0^{2\pi} \int_0^1 r\,(2-r-r)\,dr\,d\theta$$

$$= 2\pi \cdot 2 \int_0^1 (r-r^2)\,dr$$

$$= 4\pi \left(\frac{r^2}{2} - \frac{r^3}{3}\right)\Big|_0^1$$

$$= 4\pi \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{4\pi}{6} = \frac{2\pi}{3}$$