Abstract: I’ll describe a new link between geometry and probability. First, I’ll give a basic introduction to Ricci curvature, the Einstein equations and Hamilton’s Ricci flow. Next, I’ll give an introduction to Brownian motion in Euclidean space and on manifolds, and integration by parts on path space. Finally, I’ll present joint work with Aaron Naber where we discovered an infinite dimensional Bochner formula for martingales on path space, which vastly generalizes the classical Bochner formula for the heat flow on manifolds. Using these ideas, we can make sense of solutions of the Einstein equations and the Ricci flow in the setting of singular spaces, which solves a long-standing open problem.