

## Commutative Algebra Seminar

*The local cohomology of a parameter ideal with respect to an arbitrary ideal*

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**Abstract:** Let  $S$  be a complete intersection presented as  $R/J$  for  $R$  a regular ring and  $J$  a parameter ideal. Let  $I$  be an ideal containing  $J$ . It is well known that the set of associated primes of  $H^i_I(S)$  can be infinite, but far less is known about the set of minimal primes. In 2017, Hochster and Núñez-Betancourt showed that if  $R$  has prime characteristic  $p > 0$ , then the finiteness of  $\text{Ass } H^i_I(J)$  implies the finiteness of  $\text{Min } H^{i-1}_I(S)$ , raising the following question: is  $\text{Ass } H^i_I(J)$  always finite? We give a positive answer when  $i=2$  but provide a counterexample when  $i=3$ . The counterexample crucially requires  $\text{Ass } H^2_I(S)$  to be infinite. The following question, to the best of our knowledge, is open: (under suitable hypotheses on  $R$ ) does the finiteness of  $\text{Ass } H^{i-1}_I(S)$  imply the finiteness of  $\text{Ass } H^i_I(J)$ ? When  $S$  is a domain, we give a positive answer when  $i=3$ . When  $S$  is locally factorial, we extend this to  $i=4$ . Finally, if  $R$  has prime characteristic  $p > 0$  and  $S$  is regular, we give a complete answer by showing that  $\text{Ass } H^i_I(J)$  is finite for all values of  $i$ .

Wednesday, October 30 at 4:00 PM in 1227 SEO