The local cohomology of a parameter ideal with respect to an arbitrary ideal

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Abstract: Let $S$ be a complete intersection presented as $R/J$ for $R$ a regular ring and $J$ a parameter ideal. Let $I$ be an ideal containing $J$. It is well known that the set of associated primes of $H^i_I(S)$ can be infinite, but far less is known about the set of minimal primes. In 2017, Hochster and Núñez-Betancourt showed that if $R$ has prime characteristic $p > 0$, then the finiteness of $\text{Ass } H^i_I(J)$ implies the finiteness of $\text{Min } H^{i-1}_I(S)$, raising the following question: is $\text{Ass } H^i_I(J)$ always finite? We give a positive answer when $i=2$ but provide a counterexample when $i=3$. The counterexample crucially requires $\text{Ass } H^2_I(S)$ to be infinite. The following question, to the best of our knowledge, is open: (under suitable hypotheses on $R$) does the finiteness of $\text{Ass } H^{i-1}_I(S)$ imply the finiteness of $\text{Ass } H^i_I(J)$? When $S$ is a domain, we give a positive answer when $i=3$. When $S$ is locally factorial, we extend this to $i=4$. Finally, if $R$ has prime characteristic $p > 0$ and $S$ is regular, we give a complete answer by showing that $\text{Ass } H^i_I(J)$ is finite for all values of $i$. 

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