Abstract: The famous rational Hodge conjecture states that rational Hodge classes are algebraic. This long-standing heavily studied conjecture has remained widely open since it was proposed in the nineteen fifties. In contrast, certain integral analogues of the rational Hodge conjecture are known to fail in general. The main purpose of my thesis is to study how such analogues can fail and to provide new counterexamples.