Mathematics, Statistics, and Computer Science   **@ UIC**

Midwest Model Theory Seminar

Using Ultraproducts to Compare Continuous Structures

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Abstract: We revisit two research programs that were proposed in the 1960's, remained largely dormant for five decades, and then become hot areas of research in the last decade.

The monograph "Continuous Model Theory" by Chang and Keisler, Annals of Mathematics Studies (1966), studied structures with truth values in [0,1], with formulas that had continuous functions as connectives, sup and inf as quantifiers, and equality. In 2008, Ben Yaacov, Bernstein, Henson, and Usvyatsev introduced the model theory of metric structures, where equality is replaced by a metric, and all functions and predicates are required to be uniformly continuous. This has led to an explosion of research with results that closely parallel first order model theory, with many applications to analysis. In my forthcoming paper "Model Theory for Real-valued Structures", the "Expansion Theorem" allows one to extend many model-theoretic results about metric structures to general [0,1]-valued structures–the structures in the 1966 monograph but without equality.

My paper "Ultrapowers Which are Not Saturated", J. Symbolic Logic 32 (1967), 23-46, introduced a pre-ordering $\mathcal{M} \trianglelefteq \mathcal{N}$ on all first-order structures, that holds if every regular ultrafilter that saturates $\mathcal{N}$ saturates $\mathcal{M}$, and suggested using it to

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classify structures. In the last decade, in a remarkable series of papers, Malliaris and Shelah showed that that pre-ordering gives a rich classification of simple first-order structures. Here, we lay the groundwork for using the analogous pre-ordering to classify [0,1]-valued and metric structures.

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