Mathematics, Statistics, and Computer Science   **@ UIC**

Logic Seminar

Fractal Dimensions and Definability from Büchi Automata

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Abstract: Büchi automata are the natural extension of finite automata, also called finite-state machines, to a ''machine'' that accepts infinite-length inputs. We say a subset X of the reals is r-regular if there is a Büchi automaton that accepts (one of) the base-r expansions of every element in X, and rejects the base-r expansion of each element in its complement. We can analogously define r-regular subsets of higher arities of the reals, and these sets often exhibit fractal-like behavior–e.g., the Cantor set is 3-regular. There are several known–and remarkable–connections in logic to Büchi automata, including the fact that the expansion of the real additive group by every r-regular subset of [0,1] for some fixed positive integer r interprets the monadic second-order theory of the natural numbers with successor. In this talk, I will focus on some of the geometric behavior of closed r-regular set in terms of fractal dimensions, and discuss how closed r-regular sets with and without integer Hausdorff dimension form a dichotomy in terms of first order definability in expansions of the real additive group by a predicate for a specific r-regular set.

Tuesday, March 16 at 4:00 PM in Zoom