

Analysis and Applied Mathematics Seminar

The flow of polynomial roots under differentiation

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Abstract: How roots of a polynomial evolve under repeated differentiation is a classical question, which goes back to Gauss, Lucas, Marcel Riesz and many others. The dynamics can be described by a partial differential equation, formally derived by Stefan Steinerberger recently. The process is also closely related to free probability and random matrices, attracting lots of attention in the last few years.

In this talk, I will explain the intriguing relationship between the roots of trigonometric polynomials under differentiation and Steinerberger's PDE. The PDE is an active scalar equation, which is closely related to many classical models in fluid dynamics. In particular, the flow of polynomial roots bears a strong resemblance to self-organized dynamics in the modeling of animal swarms. The celebrated flocking behavior is connected to the "crystallization" phenomenon on the polynomial root distribution. I will discuss joint work with Alexander Kiselev on global wellposedness and asymptotic behavior of the PDE, as well as its rigorous connections towards the flow of polynomial roots.

Monday, April 19 at 4:00 PM in Zoom