

Analysis and Applied Mathematics Seminar

The Sharp Erdős-Turán Inequality

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Abstract: Erdős and Turán proved a classical inequality on the distribution of roots for a complex polynomial in 1950, depicting the fundamental interplay between the size of the coefficients of a polynomial and the distribution of its roots on the complex plane. Various results have been dedicated to improving the constant in this inequality, while the optimal constant remains open. In this paper, we give the optimal constant, i.e., prove the sharp Erdős-Turán inequality. To achieve this goal, we reformulate the inequality into an optimization problem, whose equilibriums coincide with a class of energy minimizers with the logarithmic interaction and external potentials. This allows us to study their properties by taking advantage of the recent development of energy minimization and potential theory, and to give explicit constructions via complex analysis. Finally the sharp Erdős-Turán inequality is obtained based on a thorough understanding of these equilibrium distributions.

Monday, October 18 at 4:00 PM in Zoom