Analysis and Applied Mathematics Seminar

Optimizing scalar transport with branching pipe flows

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Abstract: We consider the variational problem of "optimal wall-to-wall transport", in which we look to maximize the transport of a passive temperature field between hot and cold plates. Specifically, we optimize the choice of divergence-free velocity field in the advection-diffusion equation, amongst all velocities satisfying an enstrophy constraint. Previous work established an a priori upper bound to transport, scaling as the 1/3 power of the flow's enstrophy (i.e., the power used to make the flow). Recently, Tobasco and Doering '17 constructed self-similar two-dimensional branching flows saturating this bound up to an unknown logarithmic correction to scaling. We present a three-dimensional "branching pipe flow" that eliminates the possibility of this logarithmic correction, and therefore identifies the optimal scaling as a clean 1/3 power law. Our flows resemble previous numerical studies of the three-dimensional wall-to-wall problem (Motoki, Kawahara and Shimizu '18), but actually we show using a time-dependent version of our construction that the 1/3 scaling is in fact optimal in two-dimensions as well. These results have natural connections to the outstanding problem of Rayleigh–Bénard convection, and we propose several conjectures along these lines.

Monday, February 14 at 4:00 PM in 636 SEO