Combinatorics and Probability Seminar

Strict monotonicity for first passage percolation on graphs of polynomial growth and quasi-trees

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Abstract: I'll present strict monotonicity results for first passage percolation (FPP) on bounded degree graphs which either have strict polynomial growth (uniform upper and lower volume growth bounds of the same polynomial degree) or are quasi-isometric to a tree; the case of the standard Cayley graph of $\mathbb{Z}^d$ is due to van den Berg and Kesten (1993). Roughly speaking, if we use two different weight distributions to perform FPP on a fixed graph, and one of the distributions is "larger" than the other and "subcritical" in some appropriate sense, then the expected passage times with respect to that distribution exceed those of the other distribution by an amount proportional to the graph distance. If "larger" here refers to stochastic domination of measures, this result is closely related to "absolute continuity with respect to the expected empirical measure," that is, the fact that long geodesics "use all possible weights". If "larger" here refers to variability (another ordering on measures), then a strict monotonicity theorem holds if and only if the graph also satisfies a condition we call "admitting detours". I intend to sketch the proof of absolute continuity, and, if time allows, give some indication of the difficulties that arise when proving strict monotonicity with respect to variability.