Logic Seminar

The tree property up to $\aleph_{\omega^2+2}$

Dima Sinapova (Rutgers University)

Abstract: The tree property is an uncountable analogue of Konig's infinity lemma. At a cardinal κ, it states that every tree of height κ and levels of size less than κ has a cofinal branch. At $\aleph_1$ the tree property fails, and at $\aleph_2$ the tree property is equiconsistent with a weakly compact cardinal (Mitchell; Silver, 1972). Going further, in 1983 Abraham showed the tree property can hold simultaneously at $\aleph_2$ and $\aleph_3$, using a much stronger large cardinal hypothesis – a supercompact cardinal.

Since then, it has been a long standing project in set theory to obtain the tree property simultaneously at large intervals of regular cardinals. The ultimate goal being to force the tree property at every regular cardinal greater than $\aleph_1$. By a theorem of Specker a positive answer would require many failures of SCH.

We show that from large cardinals, we can force the tree property at every regular cardinal in the interval $[\aleph_2, \aleph_{\omega^2} + 2]$ with $\aleph_{\omega^2}$ a strong limit. This is joint work with Cummings, Hayut, Magidor, Neeman, and Unger.

Tuesday, April 4 at 4:00 PM in 636 SEO